International Journal of Trend in Research and Development, Volume 10(6), ISSN: 2394-9333
www.ijtrd.com

Calculus, What to Say

Rui Li
Physical Education Department of Zibo Vocational College, Shandong, Zibo, China

Abstract: Calculus has a special position in the history of scientific development and plays an important role in higher education.In the teaching process, we should pay attention to the historical background of calculus; It emphasizes the basic idea of calculus, including the idea of infinity, the transformation of curve and straight, and the unity of opposites between differential calculus and integral calculus;Through the study of calculus, we can understand the nature and characteristics of the research object of mathematics, and cultivate students' correct view of mathematical truth.

Keywords: *calculus; Infinite thought; Curved and straight; Unity of opposites; Mathematical Truth*

In the preface to the development history of the concept of calculus, the American mathematician R. Curran once commented on calculus and its teaching as follows:"Calculus, or mathematical analysis, is one of the great achievements of human thinking.Its position between natural sciences and humanities makes it a particularly effective tool for higher education.Unfortunately, the teaching method of calculus is sometimes mechanical, which can not reflect that this subject is the crystallization of a mind shaking intellectual struggle,this struggle has been going through for over 2500 years and is deeply rooted in multiple fields of human activity,and as long as people strive to understand themselves and nature for more than a day, this struggle will continue."[1] This description provides us with the following insights:First of all, calculus (or the whole mathematical discipline) is different from the humanities and cannot be reduced to natural science, but is a major discipline independent of humanities, natural science and thinking science, and becomes a link or bridge between natural science and humanities;Secondly, the special status and nature of calculus or mathematics determines that the teaching concept of calculus or mathematics is not scientism, nor humanism, but a comprehensive teaching concept integrating scientism and humanism;Third, the teaching method of calculus is neither logical nor historical, but a teaching method combining history and logic, which highlights the mental process and thinking activities of mathematicians in the process of calculus creation and invention.

From the solution of individual problems to the formation of preliminary theories, from the emergence of the second mathematical crisis to the elimination of infinitesimal paradoxes, and finally to the establishment of the theoretical basis of calculus, there are as many as dozens of mathematicians working hard.It can be said that the evolution and development history of calculus for nearly 300 years is actually the creation history of mathematicians.In this long process of exploration, it not only includes the breakthrough of mathematicians in their concepts from finite to infinite, but also experiences the confusion that it is difficult to distinguish between real infinity and latent infinity;Mathematicians not only experience the joy of using differential and integral methods to successfully solve practical problems, but also suffer from the torment of not being able to find a solution due to a lack of theoretical logic;It not only reflects the courage of mathematicians to pursue and explore truth, but also reflects their strong will to work hard and work hard;It embodies the critical spirit of mathematicians who are not afraid of authority and dare to question, as well as their belief in pursuing the generalization and unification of mathematics; Therefore, how to break through the limitations of textbook knowledge and methods, integrate the rational and humanistic spirit of mathematics, and reflect the spirit, ideas, concepts, and attitudes of mathematics, it is necessary to design an extension point of mathematical culture.In order to enrich the calculus classroom teaching content.

1.Background of Calculus

Any scientific creation not only has its objectivity and subjectivity, but also has social attributes.The social attribute of scientific creation refers to the fact that scientific creation activities cannot be separated from the external environment and the necessary conditions provided by society, such as the level of productivity development (or technological development), cultural background, and people's spiritual beliefs at that time, which all affect the process of scientific creation.Secondly, from the application of scientific research results, science is productivity, a special ideology, and a practical force that promotes social development and human civilization progress.Thirdly, whether scientific achievements are correct or not, and whether they are recognized and accepted by society, is not determined by individuals.Thomas Kuhn, the representative figure of historicism, believes that the development of science has the characteristics of paradigm shift. The new and old paradigms are incommensurable. The paradigm shift is carried out through the Scientific Revolution. The decision to accept or abandon a scientific paradigm is the result of the negotiation of the scientific community."If a proof is correct, it means that its result can enter mathematics and become a part of mathematical knowledge accepted by the mathematical community. Therefore, whether a proof is a legitimate proof should be determined by the members of the community." [2] This indicates that science (including mathematics) has strong social attributes.The simple thought of calculus can be traced back to the ancient Greek era. However, due to the low level of productivity at that time, the thought of calculus was not fully applied in production practice, lost the impetus for further development, and did not form the prototype of calculus theory.In the 16th century, the rise of the Renaissance liberated the human mind, and people began to advocate culture and pursue science, especially the development of astronomy and navigation technology, making calculus thought widely used, which is the real power of calculus.In the process of teaching calculus, emphasizing the background of calculus is to emphasize the source of calculus, emphasizing the source of calculus is to let students realize that mathematical creation is not only for personal gains and losses, interests, but also for promoting the progress of science and technology, social development and human civilization.This is also the learning goal of college students, which is the highest realm for them to pursue and explore truth.This is also the learning goal of college students, which is the highest realm for them to pursue and explore truth.

2.Basic Ideas in TheHistory of Calculus

Only the combination of mathematical knowledge and corresponding thinking methods can make mathematical knowledge have spirituality, with which comes life, and with which the mathematical knowledge of life can shine with the brilliance of human nature, only by imparting such knowledge can the classroom teaching content present a state of dialogue.The author believes that the following three basic ideas should be highlighted in the teaching process of calculus.

2.1 Infinite thought

The reason why we emphasize the idea of infinity is determined by the knowledge structure of calculus.The main content of calculus is the differentiation and integration of functions. The basic concepts of these two parts are based on the concept of limit, which is permeated with rich thinking methods. Correct understanding of the infinite idea is the key to breaking through the difficulties in learning the concept of limit.There has been a discussion about infinite thought since ancient times.The earliest thought of potential infinity in mathematics appeared in the study of the problem of “squaring the circle” by the ancient Greek sage Anaxagora.Afterwards, infinite thought formed the history of infinite development in a state of alternating development between latent infinity and real infinity.For example, Eudox and Archimedes put forward the exhaustion method by using the idea of potential infinity, and solved some special three-dimensional volumes and plane figure areas, which made the application of the idea of potential infinity in ancient times reach a peak.In the book “Zhuangzi”during the Warring States period in ancient China,There is a saying that “a pound of one foot, half of it is taken every day, and it will never be exhausted for ever.”The mathematician Liu Hui of the Three Kingdoms period also proposed the “circumcision technique” using the idea of latent infinity, and pointed out that“If the circumcision is meticulous, the loss is minimal, and if the circumcision is so profound that it cannot be cut, then it will merge with the circle without losing anything.”Democritus, the founder of atomism, held the view of real infinity.The philosopher Plato's idea of real infinity had an important impact on the later creation of calculus.The scholars who first divided infinity into potential infinity and real infinity are the ancient Greek Aristotle, and point out that potential infinity and real infinity are characterized by “Beyond eternal” and “Beyond all.”

In the initial stage of calculus, people used real infinitesimal method to solve many problems in mathematics and other technical fields, and achieved great success.Perhaps it was precisely due to Newton's insufficient understanding of infinite thought or latent infinity and real infinity at that time that serious logical contradictions arose in the process of using real infinity thought to solve problems such as velocity or tangent problems of curves, leading to the second mathematical crisis in the history of mathematical development.In order to establish the strict logic foundation of calculus, people began to accept the idea of potential infinity again.On the basis of the work of Roylier, Lacroix, Polchano, and others, mathematician Cauchy ultimately applied the idea of latent infinity to provide the concept of extreme intuitive dynamics.Weierstrass, on the other hand, uses the dialectical relationship between finiteness and infinity to deny Cauchy's view of dynamic potential infinity, reveals the internal structure of limit with the help of limited quantity and static language, and reflects the method of studying infinity with finiteness.In the process of teaching, it is helpful for students to understand and grasp the concept of limit by expounding the evolution and development history of the idea of infinity, pointing out the characteristics of potential infinity and real infinity, and comparing the differences in the methods of studying infinity in the concept of limit given by Cauchy and Weierstrass.

2.2 The transformation thought of curvatureand straightness

To judge the educational value and function of a certain ideological method, in addition to judging from the perspective of disciplinary knowledge structure, it can also examine the breadth of knowledge involved in the ideological method and its widespread application; It can also examine whether the ideological method is of great significance for students' knowledge acquisition, thinking training, and ability to solve practical problems;It can also examine whether this thinking method can guide students to develop non logical thinking methods such as analogy and association in the process of learning new knowledge, ultimately leading to analogical transfer in learning.The idea of mutual transformation between curveature and straightness is implicit in the concept of differentiation, which means that if a function is differentiable at a certain point, the increment of the y-axis of a point on the curve can be approximated as the product of the derivative of the function at that point and the increment of the independent variable.It can be intuitively seen that when the change in the independent variable is small, the curve segments within this range can be approximately replaced by corresponding tangent segments, resulting in the idea of replacing curvature with straightness or straightness with curvature.In the formation process of the concept of definite integral, the area of curved trapezoid is calculated by using the idea of replacing curve with straight. The area of each narrow curved trapezoid is approximately replaced by the area of the corresponding rectangle. The approximate value of the area of curved trapezoid is obtained through the approximate value of part of the graphic area, and then the accurate value of its area is obtained by using the limit idea.In addition, the arc length of plane curve, the volume of curved top cylinder, the second type of line integral and the second type of surface integral are all permeated with this important dialectical idea in the process of solving the cited examples.Therefore, highlighting the transformation idea of curvature and straightness, mastering the transformation mechanism of the two, can enable students to extend the process and method of solving the trapezoidal area of the curved edge analogically to problems such as the volume of the curved top cylinder, the work done by the variable force along the curve arc, and the flow rate flowing to one side of the curved surface,in order to achieve twice the result with half the effort.

2.3 The unity of opposites between differentiation and integration

In the traditional teaching process, students only realize the inverse of differential and integral operations by integrating functions first and then taking derivatives, or taking derivatives first and then integrating. This explanation can be clearly compared to the inverse of addition and subtraction, multiplication and division, which helps students understand and master.But simply revealing the connection between differential calculus and integral calculus through this point would be biased.As the internal connection between different theories, it is necessary to understand them from both a local perspective and a holistic perspective.From a historical point of view, differentiation comes from the solution of problems such as speed and tangent, while definite integral comes from the solution of problems such as distance and area.To solve the instantaneous velocity problem of variable speed linear motion, the instantaneous velocity problem is first transformed into an average velocity, which negates the local (instantaneous velocity) and transforms it into a whole (average velocity). Then, with the help of limit thinking, the average velocity is transformed into an instantaneous velocity, which negates the whole (average velocity) and transforms it into a local (instantaneous velocity). This reflects the thinking process of negation and negation from the local to the whole and then to the local.When solving the problem of variable speed linear motion, the overall distance problem is first transformed into parts, that is, the whole (the distance of the entire time period) is transformed into parts (the distance of each small time period). By calculating the approximate value of the partial distance, the approximate value of the entire distance can be obtained. Finally, the accurate value of the distance is obtained using the limit thinking, which reflects the thinking method of negation and negationfrom the whole to the local and then to the whole.The speed problem and the distance problem are reciprocal problems, and their processing methods are reciprocal, resulting in the reciprocal nature of differential and integral operations. This analysis is more helpful for students to understand the internal relationship between differential and integral calculus, and grasp the unity of opposites between these two parts.

In fact, this unified relationship can also be reasonably explained with the help of the function of the upper limit of integration and the basic formula of calculus:First, the derivative of the definite integral of the variable upper limit of a function is the function itself;Second, the calculation of definite integral is ultimately transformed into the calculation of the original function of the integrand, and the calculation of the original function is the inverse operation of differential operation;Third, with the help of the basic formula of calculus, the relationship between the integral mean value theorem and the differential mean value theorem is revealed.Analyzing the connection between differential calculus and integral calculus from multiple perspectives and levels not only enables students to have a more comprehensive and profound understanding of the inherent unity of these two different theoretical knowledge, but also helps to systematize and dynamically transform knowledge, enabling students to form a reasonable cognitive structure.

3. Cognition of Mathematics

Mathematics ontology and epistemology are two basic problems discussed in mathematical philosophy. The understanding of mathematics essentially provides a direct basis for "how to carry out mathematics teaching". Especially for first-year college students, it is more important to know mathematics through calculus teaching.

3.1 Cognition of mathematical objects

Calculus is the greatest subject after Euclid's geometry and an important milestone in the history of modern mathematics.The study of calculus provides a good opportunity for students to understand mathematics.First of all, calculus comes from the solution of practical problems. Its research objects, such as derivatives, definite integral and other concepts, have direct real prototypes, or have obvious intuitive background, which means that some mathematical objects are direct reflections of objective reality or one of its aspects.But calculus also contains research objects without obvious intuitive background. For example, the concept of continuity and uniform continuity has no real prototype, which is the result of highly abstract human thinking.In fact, since the 16th century, the research objects of the entire mathematics discipline have undergone significant changes,"More and more concepts that seem to be constantly emerging from people's minds, increasingly distant from nature, have entered mathematics."[3] This indicates that the height of mathematical abstraction has surpassed that of other natural sciences, namely the emergence of abstraction above abstraction. This object is undoubtedly established for the logical needs of the internal development of mathematics."Mathematical objects are both products of abstract thinking and have a certain degree of relative independence; they have both definite objective content and indirect and active reflections of thinking on objective reality; in addition, this objectivity and subjectivity are unified through the logical 'construction' of mathematics." [4]Understanding the characteristics of the research objects in calculus can well handle the problems of mathematics and reality, and can well understand the relationship between mathematics and logic.

3.2 Understanding the truth of mathematics

In mathematics education, the issue of mathematical truth, which should have become an important component of teaching content, is difficult to appear in traditional classroom teaching.In fact, the exploration of mathematical truth in the philosophical field is, in a general sense, an analysis and expression of human thinking patterns and methods.From the absolute truth view of mathematics to the loss of certainty and true reason in mathematics, from the logical analytical truth view to the realistic mathematical truth view, to the quasi empirical truth view of mathematics, and finally to the proposal of the mathematical model truth view, all have evolved with the development of mathematics, especially a series of revolutionary changes in mathematics in the 19th century."Mathematics reflects the illogical development of disciplines that conform to logic. Mathematics is a discipline that emphasizes logical rigor, but its development is extremely illogical." [5] The great success of calculus in practical application, the appearance of infinitesimal paradox and the foundation of calculus fully illustrate this point."If the mathematical theory (especially calculus) in the 17th and 18th centuries laid the mathematical foundation for the mechanistic view of nature, then the development of mathematics since the 20th century has fully broken through and changed the traditional concept of nature, making people's understanding of nature from the micro and macro levels reach a new height."[6]

In the teaching process, different mathematical truth views can form different teaching concepts and organize teaching activities in different teaching modes.The absolute truth view of mathematics often leads students to view mathematics as a strict discipline, and logical proof is the only way to verify mathematical conclusions. In the minds of students, mathematical knowledge is a pile of rigid and inactive knowledge.The quasi empirical truth view of mathematics believes that mathematics relies on the continuous improvement of the initial conjecture through the logic of speculation and criticism, proof and refutation, and the mathematical object is the product of logic and intuition.In the view of scholars who insist that the certainty and truth of mathematics have been lost, the truth and certainty have been lost in mathematics, but the application scope of mathematics is constantly expanding, and mathematics has gained greater creative freedom as a result, as long as mathematical research has certain theoretical or practical significance.Blindly pursuing an absolute view of truth and pursuing a single, dogmatic test standard for mathematical truth will inevitably constrain students' freedom of creation and hinder the cultivation of their creative abilities.

References

[1] Karl B. Boyer, Concept History of calculus [M]. Translated by the Mathematics Department of Shanghai Normal University. Shanghai: People's Publishing House, 1997.

[2] XiaoguiZhang. Examination of Mathematical Proof and the Absurdity of Mathematics [J].Philosophy of Science and Technology Research. 2018, 35 (4): 58-62.

[3] Klein.Translated by LijingZhang ,Jinyan Zhang.Ancient and Modern Mathematical Thought [J].Shanghai: Shanghai Publishing House,Volume 2, 109.

[4] YuxinZheng,Zeng Lin .Mathematics • Logic and Philosophy [M]. Wuhan: People's Publishing House, 1987,167-169.

[5] Zugui Zhang. On Morris Klein's Mathematical Philosophy Thought [J]. Dialectics of Nature Communication. Vol. 6, 1989, 9-19.

[6] Qin'an Huang. The Development of Mathematical Truth and Its Enlightenment on the Evolution of Natural Outlook [J]. Research on Dialectics of Nature, 2004,20 (2): 8-11.