

# A Modeling Method of Cylindrical Gear with Tooth Profile Deviation

<sup>1</sup>Dengpan Zhang, <sup>2</sup>Xianjin Liu, <sup>3</sup>Yibo Zheng and <sup>4</sup>Xian Zhao,  
<sup>1,2,3,4</sup>School of Mechanical and Power Engineering, Henan Polytechnic University, Jiaozuo, China

**Abstract---** Gear transmission system on transmission accuracy requirements are getting higher and higher, gear machining error is an important index affecting the system transmission smooth, high-precision running performance, this paper takes spur gear pair with machining error as the research object, proposes a finite element modeling of straight cylindrical gear with tooth shape deviation, studies the influence of different machining methods on tooth surface deviation, and analyzes the influence of tooth surface deviation on gear transmission dynamics. The machining error was parameterized, the gear pair was digitally modeled, the contact finite element analysis method was used to study the dynamic characteristics of gear transmission performance, and the effect and influence of tooth surface deviation on internal excitations such as gear transmission error, meshing stiffness and meshing impact were analyzed. The results show that the modeling method has certain reference value for the analysis of gear transmission with tooth surface deviation, and can provide a basis for the design of gear transmission system.

**Keywords---** Spur gear; Tooth Surface deviation; Finite Element; Dynamics

## I. INTRODUCTION

In the manufacturing process of gears, the machining of gears is an important link that affects the smooth transmission and high-precision operation of the gear system. To ensure the smooth operation of the gear system under normal working conditions, it is necessary to reduce the internal excitation during the gear transmission process as much as possible. In the actual machining process of gears, machining errors are inevitable, which makes vibration and noise during gear operation, transmission errors and meshing stiffness will be greatly affected. Therefore, it is of practical significance to study the influence of gear machining error on gear transmission characteristics.

In terms of the machining error of gears, many scholars at home and abroad have carried out relevant research. Liu Chang et al. established an error tooth surface model based on the distribution law of gear machining error, and studied the research law of machining error on the internal excitation of spur gear pair[1]. Wang Xunlang et al. comprehensively considered the machining error of gear teeth, shaft deflection and tooth side clearance, and established the transmission error function of the gear transmission chain[2]. Wu Yang et al. based on the principle of cylindrical gear forming of circular arc tooth line, established the structure model of gear machine tool and the coordinate system system of the machine tool, and studied the calculation method of gear error surface[3]. Li Xiaozhen et al. derived the equation of the tooth profile of the surface gear and the equation of the contact line of the face gear with machining error, and studied the influence of tool

offset and intersection error on the contact trajectory of the face gear tooth[4]. Cao Congqing et al. analyzed the kinematics of gears with machining errors and gears without machining errors, and compared the effects of machining errors on the angular velocity of the driven wheel and the contact force between the teeth[5]. Tong Cao et al. considered the dynamic performance of gears under the action of installation error and pitch deviation, and studied the influence of their joint action on gear transmission[6]. Shi Zhaoyao et al. considered the overall error concept of gear pair, and studied the influence of the meshing stiffness of the gear teeth and the machining error on the vibration response of the gear system under different working conditions[7]. Tengjiao Lin et al. comprehensively analyzed the effects of machining error, assembly error and gear tooth modification on helical gear transmission error by using finite element modeling[8]. Min Zhang et al. comprehensively consider the machining and assembly errors of key components such as shafts, gears, and housings, and propose a mathematical method for predicting transmission errors of gear system based on design tolerances to find out the key components and factors affecting the assembly performance of gear systems[9].

In the study of finite element analysis, many scholars often cannot accurately model gears with tooth shape deviation when analyzing machining errors. To solve this problem, a modeling method for tooth shape deviation of cylindrical gears is proposed, and the influence of tooth surface deviation on gear transmission performance is studied and analyzed.

## II. TYPES OF MACHINING ERRORS

The gear machining error mainly comes from the cycle error and tool error of the machine tool transmission chain, and the machining error that has a great influence on the transmission of the gear system mainly includes tooth pitch deviation and tooth shape deviation.

1) Tooth shape deviation: tooth shape deviation, also known as tooth profile deviation, refers to the difference between the actual tooth shape and the designed tooth shape. This error is measured in the end plane as the minimum normal distance between the two tooth profiles (actual and design) whose length is perpendicular to the involute tooth shape direction within the end face. This error is caused by the difference between the actual machined tooth profile and the design tooth profile that is not as smooth as ideal, and the uneven state is not ideal. Therefore, tooth profile error is a difference in shape. As shown in Fig.1, the working part of the actual tooth profile is included in the scope of the designed tooth profile.

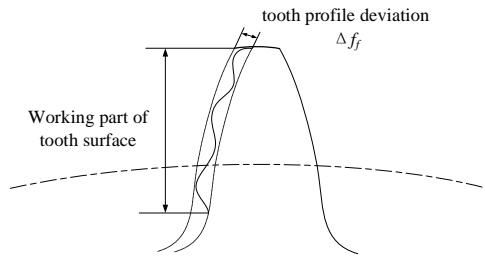


Fig.1 Definition of profile deviation

2) Tooth pitch deviation: Tooth pitch error refers to the difference between the actual tooth pitch and the theoretical tooth pitch on the indexing circle. As shown in Fig.2, it represents the position deviation of the tooth profile in the circumferential direction and affects the ideal meshing position. The essence of this error is the deviation between the position of the actual tooth profile and the position of the theoretical tooth profile, which in turn destroys the meshing state of the gear pair.)

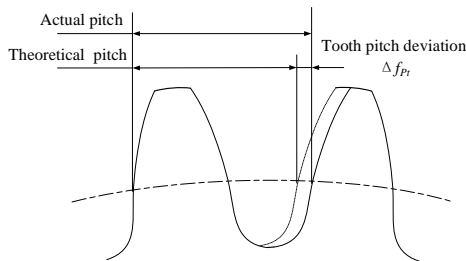


Fig.2 Definition of pitch deviation

Comprehensive analysis of tooth pitch deviation and tooth shape deviation shows that the general machining error can be regarded as a synthesis of the above two deviations, as shown in Fig.3. Because the gear model with tooth pitch deviation is relatively easy to obtain during modeling, this paper mainly models and analyzes the gear pair with tooth shape deviation, and proposes a new modeling method.

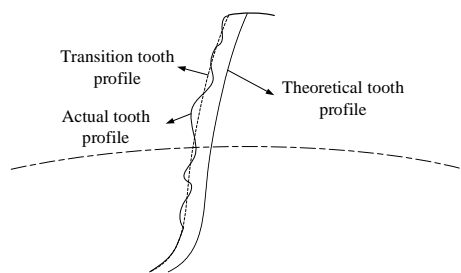


Fig.3 Definition of machining error

### III. CONSTRUCTION AND SOLUTION OF FINITE ELEMENT MODELS

#### A. Involute mathematical model with tooth surface deviation

The standard tooth profile involute of the gear is the basis for ensuring the smooth transmission of the gear, and the standard involute equation is established according to the geometric relationship between the involute and the base circle shown in Fig.4

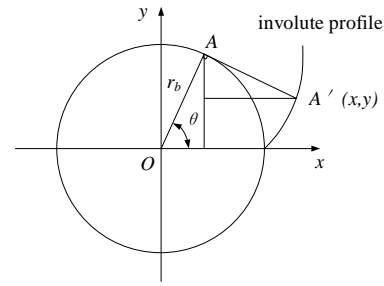


Fig.4 Geometric relationship between involute and base circle

$$\begin{cases} x = r_b \cos \theta + \theta r_b \sin \theta \\ y = r_b \sin \theta - \theta r_b \cos \theta \end{cases} \quad (1)$$

where  $r_b$  is the radius of the base circle and  $\theta$  is the sum of the spread angle and pressure angle at any point on the involute.

The radius of the gear base circle  $r_b$ , the radius of the tooth top circle  $r_a$ , and the radius of the tooth root circle  $r_f$  are respectively expressed as

$$\begin{cases} r_b = \frac{1}{2} m z \cos \alpha \\ r_a = \frac{1}{2} m z + h_a^* m \\ r_f = \frac{1}{2} m z - (h_a^* + c^*) m \end{cases} \quad (2)$$

where  $m$  is the gear modulus,  $z$  is the number of teeth,  $\alpha$  is the pressure angle, and  $h_a^*$  and  $c^*$  are the tooth top height coefficient and the top clearance coefficient, respectively.

In the actual machining process of the gear, due to vibration and other factors, the entire tooth shape curve is uneven, and the shape of the tooth profile changes. At this point, it can be assumed that a certain range of random errors are generated in the  $x$  and  $y$  directions, and the error term is added after formula (1) to obtain an involute expression with tooth shape deviation:

$$\begin{cases} x = r_b \cos \theta + \theta r_b \sin \theta \\ y = r_b \sin \theta - \theta r_b \cos \theta + \delta \sin(\lambda \theta) \end{cases} \quad (3)$$

In the formula, the value of the  $\delta$  is related to the amount of tooth surface deviation, representing the amplitude of the tooth surface deviation.  $\lambda$  indicates the size of fluctuation period of tooth surface deviation.

#### B. Build a gear finite element model with machining error

The geometric model of the spur gear pair is established, and the parameters are shown in Table.1. In order to reduce the number of finite element meshing, the three-dimensional model of the spur cylindrical gear pair is established, and the inner diameter of the gear shaft bore is 40mm, as shown in Fig.6.

Tab.1 Main parameters of cylindrical straight gear pair

Parameter	Gear A	Gear B
Number of teeth	20	20
The modulus	3	
Pressure Angle	20	
Tooth width	10	
Modification coefficient	0	

Enter an involute expression with tooth profile deviation in the Croe software, and generate an involute tooth profile with tooth profile deviation from the involute expression with tooth profile deviation, as shown in Fig.5. Besides, Fig.5 partially enlarges the involute tooth profile, and it can be seen that the involute is uneven, which is more in line with the surface morphology characteristics of gears with machining errors.

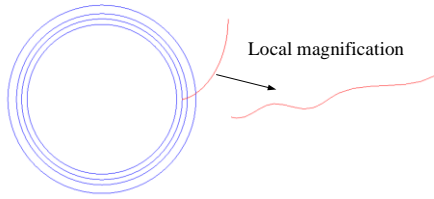


Fig.5 Involute tooth profile with tooth shape deviation

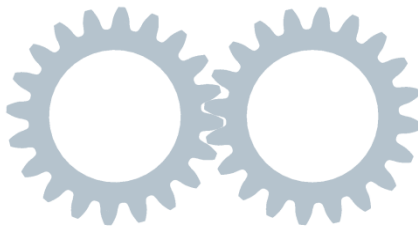


Fig.6 Geometry of gear pair with machining error

The gear pair geometry model with machining errors is imported into ANSYS for preprocessing, and the finite element model of gear pairs with machining errors is obtained, as shown in Fig.7.

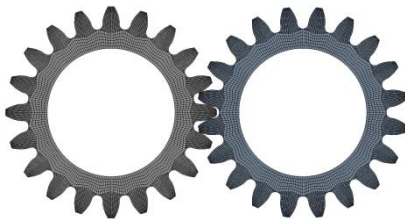


Fig.7 Finite element model of gear pair with machining error

### C. Mathematical model of spur cylindrical gears

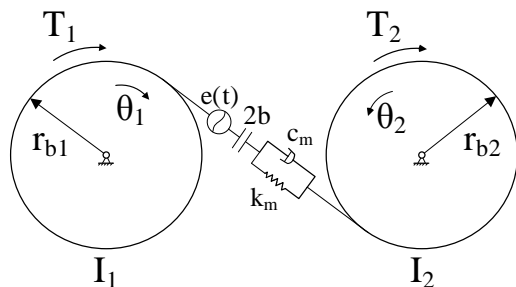


Fig.8 Torsional vibration model of straight gear pair

Simplify the gear system to a torsional vibration model [10], as shown in formula 8. According to the balance of force and torque of each gear, the equation of motion of the gear pair can be expressed as:

$$\begin{cases} I_1 \ddot{\theta}_1 + r_{b1} c_m \delta(t) + r_{b1} k_m f[\delta(t)] = T_1 \\ I_2 \ddot{\theta}_2 - r_{b2} c_m \delta(t) - r_{b2} k_m f[\delta(t)] = -T_2 \end{cases} \quad (4)$$

In the formula, the subscripts 1,2 represent the drive and driven wheels, respectively,  $I$  is the moment of inertia,  $\theta$  is the deflection angle of the gear,  $r_b$  is the radius of the base circle,  $c_m$  is the meshing damping,  $k_m$  is the time-varying meshing stiffness,  $e(t)$  is the gear pair meshing comprehensive error,  $T_1, T_2$  are the external load moments acting on the drive and driven gears, respectively.

The expression for the transmission error is:

$$\delta(t) = r_{b1} \theta_1(t) - r_{b2} \theta_2(t) - e(t) \quad (5)$$

The gear tooth meshing comprehensive error  $e(t)$  is expressed as:

$$e(t) = e_0 \cos(\omega_e t) \quad (6)$$

where  $e_0$  is the fluctuation term and  $\omega_e$  is the meshing frequency

Subtracting the two equations in formula (4) yields:

$$m_e \ddot{q} + c_m \dot{q} + k_m q = f_m - m_e \ddot{e}(t) \quad (7)$$

where  $m_e$ ,  $c_m$  and  $k_m$  are gear mass, meshing damping and meshing stiffness, respectively, and  $f_m$  is the excitation force.

In the transient dynamics model, inertial forces and damping can be ignored in a quasi-stationary environment, which can be considered  $e(t) = 0$ , and the expression of the meshing stiffness of the gear teeth can be described as:

$$k_m(t) = \frac{T_1}{(r_{b1} \theta_1 - r_{b2} \theta_2) r_{b1}} \quad (8)$$

### D. Finite element analysis results and post-processing

As can be seen from the gear contact stress cloud in Fig.9, the contact type of the gear teeth is two small plane contacts. Theoretically, when the gear rigidity is large enough, the gear will not deform, and the meshing tooth surface is line contact; However, in the actual meshing process, due to the action of force, the gear teeth will be deformed, and the tooth surface contact will change from line contact to small plane contact. Through finite element analysis, the rotation angles of large and small gears are extracted separately, and the transmission error and time-varying meshing stiffness of spur cylindrical gears can be calculated by formula (5) and formula (8).

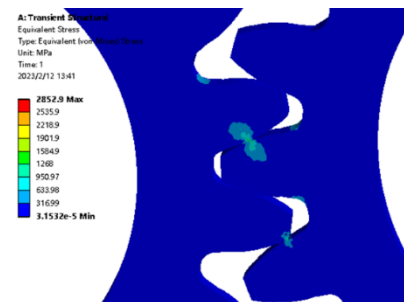


Fig.9 Cloud Chart of Gear Contact Stress

## IV. ANALYSIS OF SIMULATION RESULTS

In this paper, by improving the standard involute tooth profile of spur cylindrical gear, the involute tooth profile with tooth shape deviation is obtained, and the finite element model of spur cylindrical gear pair with tooth shape deviation is established, and the influence of tooth shape deviation on the

time-varying meshing stiffness, transmission error and meshing impact force of the gear is analyzed, as shown in Fig.10-12.

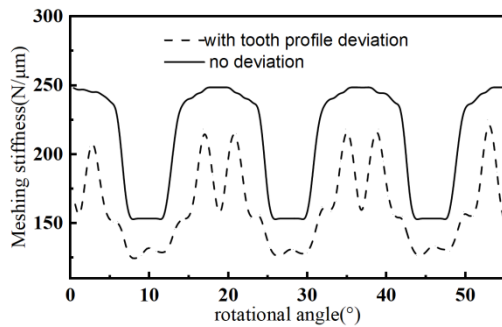


Fig.10 Variable meshing stiffness when gearing

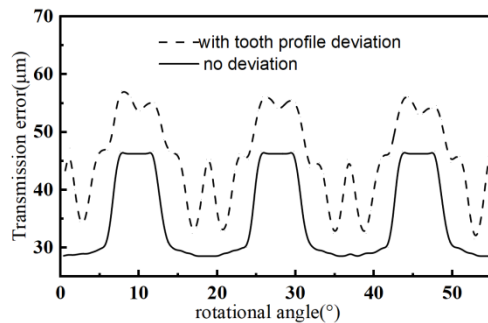


Fig.11 Gear transmission error

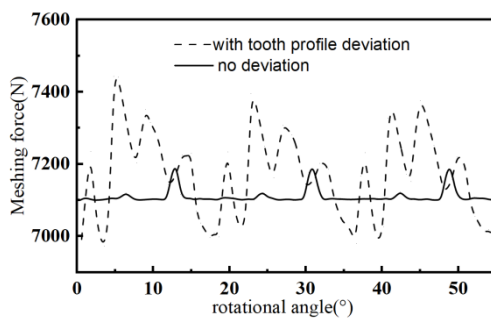


Fig.12 Gear tooth meshing force

It can be seen from the time-varying meshing stiffness of the gear teeth that when the gear pair with tooth shape deviation meshes, the meshing stiffness of the gear teeth is greatly reduced as a whole compared to the gear pair without deviation. In the process of alternating the single and double teeth of the gear, although the gear pair with tooth shape deviation also shows periodic changes, the meshing stiffness suddenly drops sharply during the meshing process of the double teeth, which will cause the gear to have transmission loss, thereby causing a sharp change in transmission error, aggravating the vibration and noise of the gear, and reducing the service life of the gear system. It can be seen from Fig.11 that compared with the gear transmission error without deviation, the peak-to-peak value of the gear pair transmission error with tooth shape deviation is larger, and the amplitude of the transmission error changes more frequently with the rotation of the gear system, which also corresponds to the time-varying meshing stiffness in Fig.10. It can be seen from the gear tooth meshing force in Fig.12 that the gear system without deviation will have a sudden change in the meshing force only when the single and double teeth are alternately meshed during the transmission process, that is, meshing and meshing impact, while the gear pair with tooth shape deviation has a large fluctuation in the meshing force of the gear teeth during the

transmission process, and the frequency of fluctuations is higher, which will make the meshing impact of the gear teeth more frequent and the meshing impact force is also greater.

## CONCLUSION

Using the gear system modeling method with tooth shape deviation proposed in this paper, it can be simulated and analyzed that the straight cylindrical gear with tooth shape deviation will be reduced as a whole compared with the standard involute spur gear when transmission, in addition to the change of meshing stiffness in the alternating process of single and double teeth, the gear system with tooth shape deviation may also have a large mutation in the process of single or double tooth meshing. With the rotation of the gear system, the amplitude of the transmission error changes more frequently, so the tooth shape deviation has a great impact on the gear transmission error, and when high-precision transmission is required, the tooth shape deviation should focus on reducing the tooth shape deviation. When marking involute straight tooth cylindrical gear, only when the single and double teeth are alternately meshed, there will be a sudden change in the meshing force, that is, meshing and meshing impact, and the gear pair with tooth shape deviation has a large fluctuation in the meshing force of the gear teeth during the transmission process, and the frequency of fluctuations is higher, which will make the meshing impact of the gear teeth more frequent and the meshing impact force is also greater.

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