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A Study on Closure Operations α-Ary in Multiple Channels of Digital Topology

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Abstract - The paper is about a certain generalized topological structures, with the aim of showing that can provide suitable framework within which to compare various approaches to digital topology with an ordinary topology. Here mainly speaks about the generalized topological structures, called closure operations, that are associated with α-ary relations $(\alpha > 1)$. The closure operations which is commonly used as a basic topological structure in digital topology. Some of these structures are well-behaved with respect to connectedness and so are suitable for applications in digital topology. Mainly the multiple express channels increase throughput to saturate wire density in α-ary n-cube, where n stands for natural numbers.

Keywords: Homotopy, α-ary, Digital Topology, Closure Operation, m-Dimensional.

I. INTRODUCTION

Digital topology was first studied in the late 1960s by the computer image analysis researcher Azriel Ronsenfeld (1931-2004), whose publications on the subject played a major role in establishing and developing the field. The term "*digital topology*" was invented by Rosenfeld, who used it in a 1973 publication for the first time. A related work called the grid cell topology appeared in Alexandrov-Hopf's book topology can be considered as a link to classic combinatorial topology.

Digital topology deals with properties and features of two-dimensional (2D) or three-dimensional (3D) digital images that correspond to topological properties (e.g., connectedness) or topological features of objects. It mainly study of geometric and topological properties of digital images. It proved to be a useful tool for solving some problems of computer graphics and image processing. Despite its name, the theory was originally based on utilizing graph-theoretic rather than topological methods [3, 1, 4, 5]. It was only in the late 1980s that a topological approach to digital topology was used for the first time in [9].

Digital image is an image that has been converted into binary array which is readable on a computer or some other digital device. Digital image analysis involves "segmenting" the image into parts and find the relationship between these parts and studied various properties among them. Digital topology studies the properties of this set of pixel that correspond to topological properties of the original object. The feature of digital topology lies in analysis of the properties like connectedness, adjacency, digital homotopy etc. with different fields like pattern recognition, medical imaging, image processing, neuroscience, geo-science etc. Closure operations that are associated in a special way with α-ary relations $(\alpha > 1)$ were introduced in [2] and then studied also in [10]. All closure operations studied are associated with α-ary relations ($\alpha > 1$ ordinals) and this fact is used with advance in our topological investigations.

Express cubes are α-ary n-cubes interconnection networks augmented by express channels that provide a short path for nonlocal messages. An express cube combines the logarithmic diameter of a multistage network with the wire-efficiency and ability to exploit locality of a low-dimensional mesh network. The insertion of express channels reduces the network diameter and thus the distance component of network latency. Wire length is increased allowing networks to operate with latencies that approach the physical speed-of-light limitation rather than being limited by node delays. Express channels the wire bisection in a manner that allows the bisection to be controlled independent of the choice of radix, dimension, and the channel width. By increasing wire bisection to saturate the available wiring media, throughput can be substantially increased. No changes are required for local communication controllers.

Interconnection networks are applied to pass messages containing data and synchronization information between the nodes of concurrent computers [6], [7], [8]. An interconnection network is characterized by its *topology*, routing, and flow control. The topology of a network is the arrangement of its nodes and channels into a graph. Routing determines the path chosen by a message in this graph. Flow control deals with the allocation of channels and buffer resources to a message as it travels along this path this paper only deals with topology.

A. Topology

A *topology* on a set X is a collection τ of subsets of X having the following properties,

1. $\emptyset$ and X are in τ
2. The union of the elements of any sub collection of τ is in τ.
3. The intersection of the elements of any finite sub collection of τ is in τ.

A set X for which a topology has been specified is called *topological space.*

B. Digital Topology

Digital topology deals with properties and features of two-dimensional (2D) or three-dimensional (3D) digital images that correspond to topological properties (e.g., connectedness) or topological features of objects.

C. Closure Of A Set

The *closure* of a set S is the set of all points of closure of S, that is, the set S together with all of its limit points. The closure of S is denoted $cl(S), CL(S), \bar{s}$ or s^{-}. The closure of a set has the following properties.

- $cl(S)$ is a closed superset of S.
- $cl(S)$ is the intersection of all closed sets containing S.
- $cl(S)$ is the smallest closed set containing S.
- $cl(S)$ is the union of S and its boundary $\partial(S)$.
- A set S is closed if and only if $S= cl(S)$.
- If S is a subset of T, then $cl(S)$ is a subset of $cl(T)$.
- If A is a closed set, then A contains S if and only if A contains $cl(S)$.

D. Closure Operation

Let X be a set, we denote by exp X its power set, i.e., the set of all subsets of X. By a *closure operation u* on a set X we mean a map u: exp $X \rightarrow$ exp X fulfilling $u\emptyset = \emptyset$, $A \subseteq X \Rightarrow A \subseteq uA$, and $A \subseteq B \subseteq X \Rightarrow uA \subseteq uB$.

II. PROPOSITIONS

A.Proposition 1:

For any α-ary relation R on a set X, (X, u_R) is an $S_{\langle\alpha\rangle}$-space.

Proof:

Let $A \subseteq X$ and $x \in u_R A$.

If $x \in A$, then by definition. We get,

$x \in u_R\{x\} \subseteq$

$\bigcup\{u_R B; B \subseteq A \text{ and card } B < \langle\alpha\rangle\}$

.

Suppose $x \notin A$. Then there exist, $(x_i |\ i < \alpha) \in R$ and i_0, $0 < i_0 < \alpha$, such that $x = x_{i_0}$ and $x_i \in A$ for all $i < i_0$.

Hence $\{x_i |\ i < i_0\} \subseteq A$ and card$\{x_i |\ i < i_0\} \leq$ card i_0.

But card $i_0 < \alpha = \langle\alpha\rangle$ whenever α is a cardinal and card $i_0 \leq$ card $\alpha < \langle\alpha\rangle$ whenever α is not a cardinal.

Since $x \in u_R\{x_i |\ i < i_0\}$, we have $x \in \bigcup\{u_R B; B \subseteq A$ and card $B < \langle\alpha\rangle\}$.

Therefore the inclusion $u_R X \subseteq \bigcup\{u_R B; B \subseteq A$ and card $B < \langle\alpha\rangle\}$ is valid.

B. Preposition 2:

(Z, u_n) is a connected closure space.

Proof:

Let ω denote the least infinite ordinal and let $(B_i |\ i < \omega)$be the sequence given by $B_i = A_{i/2}$ whenever i is even and $B_i = A_{-(i+1)/2}$ whenever i is odd.

(i.e) $(B_i |\ i < \omega) = (A_0, A_{-1}, A_1, A_{-2}, A_2, \cdots)$.

Then B_i is connected in (Z, u_n) for each $i < \omega$.

For each $l \in Z$ there holds $A_l \cap A_{l+1} = \{l(n-1)\} \neq \emptyset$.

Thus, we have $B_0 \cap B_1 \neq \emptyset$.

Let i_0 be a natural number with $i_0 > 1$.

Then $B_{i_0} \cap B_{i_0 - 2} \neq \emptyset$ because $B_{i_0} = A_{i_0/2}$ and $B_{i_0-2} = A_{(i_0/2)-1}$ whenever i_0 is even.

And $B_{i_0} = A_{-(i_0+1)/2}$ and $B_{i_0-2} = A_{-(i_0+1)/2+1}$ whenever i_0 is odd.

Hence $\bigcup_{i<i_0} B_i \cap B_{i_0} \neq \emptyset$ for each $i_0, 0 < i_0 < \omega$.

Therefore $\bigcup_{i<\omega} B_i$ is connected.

But $\bigcup_{i<\omega} B_i = \bigcup_{l\in Z} A_l = Z$, which proves the statement.

III. MULTIPLE CHANNEL IN TOPOLOGY

Digital topology and geometry refer to the use of topologic and geometric properties and features, e.g., straightness, convexity, compactness, distance, adjacency, boundary, connectivity, components, paths etc for image defined in digital grids. It is widely used in medical imaging applications is to assess the function, physiology, abnormality of internal human organs or tissues. Overview of the role topology plays a digital image processing. It is mainly used in TV, 3D images, Photo editor, medical, FM, Channel capacity etc. Now the express cubes improving the performance of α-ary n-cube inter connection networks. Express cubes can be applied independent of routing and flow control strategies. The messages may be sent between the processing nodes a message passing multi computer or between the processors and memories of a shared memory multiprocessor. An inter connection network is characterized by its topology, routing, and flow control.

IV. NOTATIONS

The following symbols are used in this application. They are listed here for reference.

i, the number of nodes between interchanges in an express cube.

M, the number of express channels through each node.

n, the dimension of the network.

W, the width of a channel in bits.

w, the width of a node-the number of wires that may pass into a node in each dimension.

k, the radix of the network-the length in each dimension.

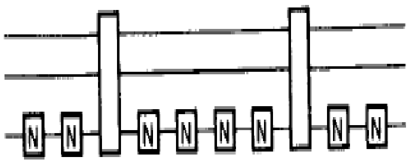
V. MULTIPLE EXPRESS CHANNELS INCREASE THROUGHPUT TO SATURATE WIRE DENSITY

To first order, network throughput is proportional to wire bisection and hence wire density. If more wires are available to transmit data across the network, throughput will be increased provided that routing and flow control strategies are able to profitably schedule traffic onto these wires. Many regular network topologies, such as low-dimensional α-ary n-cubes, are unable to make use of all available wire density because of pin limitations. The wire bisection of an express cube can be controlled independent of the choice of radix α, dimension n, or channel width W by adding multiple express channels to the network to match network throughput with the available wiring density w.

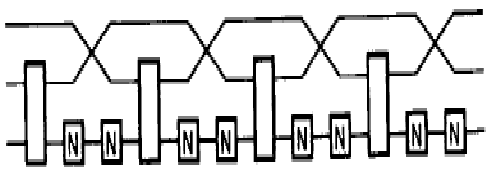
Fig.1 shows two methods of inserting multiple express channels. Multiple express channels may be handled by each interchange as shown in Fig. 1.1 (a). Alternatively, simplex interchanges can be interleaved as shown in the Fig.1 (b).

In method (a), using multiple channel interchanges, an interchange is inserted every i nodes as above and each interchange is connected to its neighbors using m parallel express channels. Fig. 1(a) show a network with $i = 4$ and $m = 2$. The interchange acts as a concentrator combining messages arriving on the m outgoing express channels. This method has the advantage of making better use of the express channels since any message can route on any express channels. Flexibility in express channel assignment is achieved at the expense of higher pin count and limited expansion.

With method (b), interleaving simplex interchanges, m simplex interchanges are inserted into each group of i nodes. Each interchange is connected to the corresponding interchange in the next group by a single express channel. All messages from the nodes immediately before an interchange will be routed on that interchange's express channels. Because load cannot be shared among interleaved express channels, an uneven distribution of traffic may result in some channels being saturated while parallel channels are idle. Method (b) has the advantage of using simple interchanges and allowing arbitrary expansion. In the extreme case of inserting an interchange between every pair of nodes the resulting topology is almost the same as the topology that would result from doubling the number of dimension.



(a)



(b)

Figure. 1 Multiple express channels allow wire density to be increased to saturate the available wiring media. Express channels can be added using either

a) Interchange with multiple express channels, or
b) Interleaved simplex interchanges.

Both of the methods illustrated in Figure.1 have the effect of increasing the wire density by a factor of $m + 1$. To first order, network throughput will increase by a similar amount. There will be some degradation due to uneven loading of parallel channels.

The use of multiple express channels offsets the load imbalance between express and local channels. If traffic is uniformly distributed, the average fractions of messages crossing a point in the center of the network on a local channel is $f_0 = 2i / k$ as compared to $f_1 = (k - 2i) / k$ crossing on an express channel. For large networks where $k \gg i$, the bulk of the traffic is on express channels. Increasing the number of express channels applies more of the network bandwidth where it is most needed.

Multiple express channels are an effective method of increasing throughput in networks where the channels width is limited by pin out constraints. For example, in the J-machine the channel width W = 9 is set by pin limitations. The printed-circuit board technology is capable of running w = 80 wires in each dimension across the 50 mm width of a node. Even with many of these wires used for local connections, four parallel 15-bit wide channels can be easily run across each node. A multiple express channel network with m = 3 could use this available wire density to quadruple the throughput of the network.

CONCLUSION

Thus the suitable framework within which to compare various approaches to digital topology with an ordinary topology that are associated α-ary relations (α >1) are well-behaved with respect to connectedness in digital topology. For any natural number$n > 1$, a closure operation on $Z \times Z$ which is obtained as a product of two copies of Z with special n-ary relation on Z. To show that an n-ary digital m-dimensional space is useful for solving problems of digital topology, we have to verify that it behaves, in a certain sense, like the real of the m-dimensional (topological) space. The binary digital plane has been investigated in many papers and there are also some papers devoted to the study of the binary digital sets 3-dimensional space.

Express cube are α-ary n-cubes augmented by express channels that provides a short path for nonlocal message. An express cube retains the wire efficiency of a conventional α-ary n-cubes while providing improved latency and throughput that are limited only by the wire delay and available wire density. Multiple express channels can be used to increase throughput to the limit of the available wire density. The express cube combines the low diameter of multistage interconnection networks with the wire efficiency and ability to exploit locality of a low-dimensional mesh network.

Express channel are added to α-ary n-cubes by periodically inserting interchanges into each dimension. No modifications are required to the routers in each processing node; express channels can be added to most existing α-ary with n-cubes network. Interchanges allow the wire density, speed, and signaling levels to be changed by module boundaries.

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