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Strongly Pseudo-Regular and Strongly Pseudo-Normal Topological Spaces

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Abstract: In this paper strongly pseudo-regular and strongly pseudo-normal topological spaces have been introduced and their properties have been studied. A number of important theorems regarding these spaces have been established.

Keywords: Strongly pseudo-regular spaces, strongly pseudo-normal spaces, compact set, Hausdorff spaces, equivalence relation, projection mapping.

Mathematics Subject Classification: 54D10, 54D15, 54A05, 54C08.

1. INTRODUCTION

This is the second in a series of our papers on generalizations and specializations ofregular and normal topological spaces. The first such paper has appeared in 2018[17]. Earlier,regular and normal topological spaces have been generalized in various other ways. p-regular, p-normal, β -normal and γ -normal spaces ([7], [8], [10], [12], [15]) are several examples of some of these.

Here we have introduced stronglypseudo-regular and stronglypseudo-normal spaces and studied their important properties. Manyimportant results about these spaces have beenestablished.

We have used the terminology and definitions of text books ofS. Majumdar and N. Akhter [1], Munkres [2], Dugundji [3], Simmons [4], Kelley [5] and Hocking-Young[6].

Unless otherwise stated, every compact set considered in this paper will have at least two elements.

II. PRELIMINARIES

We start with the definitions of γ -normal, p -normal, β -normal spaces.

A subset A of a topological space X is said to be **regular open** (resp. **regular closed**) if A=int(cl(A)) (resp. cl(int(A)),**preopen** (briefly **p-open**) if A $\subseteq$ int(cl(A)), β **-open** if A $\subseteq$ cl(int(cl(A))), γ **-open** if A $\subseteq$ cl(int(A)) $\cup$ int(cl(A)).

Definition 2.1:A topological spaces X is said to be γ **-normal** (resp. **p-normal**, β **-normal** [7]) if for every pair of disjoint closed subsets A and B of X, there exist disjoint γ -open (resp. p-open, β -open) sets U and V of X such that A $\subseteq$ U and B $\subseteq$ V.

We shall now define and study strongly pseudo-regular spaces as specializations of pseudo regular spaces (see [17]).

III. STRONGLY PSEUDO-REGULAR SPACES

Definition 3.1: A topological space X will be called**strongly pseudo-regular** if, for each compact set K and for every $x \in X$ with $x \notin K$, there exist open sets G and H such that $x \in G$ and $K \subseteq H$ and $\overline{G} \cap \overline{H} = \phi$.

Ex 3.1: X= ℝ with usual topology is strongly pseudo-regular. To see this, let K be a non-empty compact subset of X and let $x \in X,\ x \notin K$. Then, by Heine Borel Theorem, K is closed and bounded. Hence, K may be written as K= $\bigcup_{i=1}^{\infty}[a_i, b_i]$, where $[a_i, b_i] \cap [a_j, b_j] = \phi$ if $i \neq j$.

Let $a_{i_0} = \min_i\{a_{i_0}\}$, $b_{j_0} = \max_j\{b_{j_0}\}$. Since $x \notin K$, one of the following three conditions must hold:

(i) $x < a_{i_0}$

(ii) $x > b_{j_0}$

(iii) there exist a_{j_1}, a_{k_1} and b_{j_1}, b_{k_1} such that $a_{j_1} \leq b_{j_1} < a_{k_1} \leq b_{k_1}$, and [a_{j_1} , b_{j_1}] and [a_{k_1} , b_{k_1}] are consecutive intervals in K, and $b_{j_1} < x < a_{k_1}$.

If (i) holds, let $\partial_1 = \frac{1}{3}|x - a_{i_0}|$, and let $U_1 = (x - \partial_1, x + \partial_1)$, $V_1 = (a_{i_0} - \partial_1, b_{j_0} + \partial_1)$. Then U_1, V_1 are open and $\overline{U_1} \cap \overline{V_1} = \phi$. Also $x \in U_1$, $K \subseteq V_1$.

If (ii) holds, let $\partial_2 = \frac{1}{3}|x - b_{j_0}|$, and let $U_2 = (x - \partial_2, x + \partial_2)$, $V_2 = (a_{i_0} - \partial_2, b_{j_0} + \partial_2)$. Then U_2, V_2 are open, $x \in U_2$, $K \subseteq V_2$ and $\overline{U_2} \cap \overline{V_2} = \phi$.

If (iii) holds, let $\partial_3 = \frac{1}{3}\min\{x - b_{j_1}, a_{k_1} - x\}$, and let $U_3 = (x - \partial_3, x + \partial_3)$ and $V_3 = (a_{i_0} - \partial_3, b_{j_1} + \partial_3) \cup (a_{k_1} - \partial_3, b_{j_0} + \partial_3)$. Then U_3, V_3 are open, $x \in U_3$, $K \subseteq V_3$ and $\overline{U_3} \cap \overline{V_3} = \phi$.

Thus, X is strongly pseudo-regular.

Theorem 3.1: Every strongly pseudo-regular space is pseudo regular but the converse is not true in general.

Proof: The first part is obvious. To prove the converse, let X={a,b,c,d} and $\Im = \{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}\}$. Then (X, $\Im$) is a topological space. The closed subsets of X are $X, \phi, \{b,c,d\}, \{a,d\}, \{d\}$. Let K= $\{a\}$. Then K is compact and $b \notin K$ .Then we have open sets G= $\{a\}$, H= $\{b,c\}$ such that $K \subseteq G$, $b \in H$ and $G \cap H = \phi$. Hence X is pseudo regular.G and H are the only disjoint open sets which contain K and b respectively.

Now, we have $\overline{H} = \{b,c,d\}$, $\overline{G} = \{a,d\}$ and $\overline{G} \cap \overline{H} = \{d\} \neq \phi$. Hence X is not strongly pseudo-regular.

Theorem 3.2: Any subspace of a strongly pseudo-regular space is strongly pseudo-regular.

Proof: Let X be a strongly pseudo-regular space and $Y \subseteq X$. Let $y \in Y$ and K be a compact subset of Y such that $y \notin K$. Since K is compact in Y, so K is compact in X. Since X is strongly pseudo-regular, there exist open sets G and H of X such that $y \in G$ and $K \subseteq H$ and $\overline{G} \cap \overline{H} = \phi$. Let U= $G \cap Y$ and V= $H \cap Y$. Then U and V are open sets of Y where $y \in U$ and $K \subseteq V$ and $\overline{U} \cap \overline{V} = \phi$. Hence Y is strongly pseudo-regular.

Corollary 3.1: Let X be a topological space and A, B are two stronglypseudo-regular subspace of X. Then $A \cap B$ is strongly pseudo-regular.

Proof: Since $A \cap B$ being a subspace of both A and B, $A \cap B$ is strongly pseudo-regular by the above theorem.

Theorem3.3: A topological space X is strongly pseudo-regular if,for each $x \in X$ and for any compact set K not containing x, there exists an open set H of X such that $x \in H \subseteq \overline{H} \subseteq K^c$.

Proof: Let X be strongly pseudo-regular and let K be compact in X. Let $x \notin K$ i.e; $x \in K^c$. Since X is strongly pseudo regular, there exist open sets U, V such that $x \in U, K \subseteq V$ and $\overline{U} \cap \overline{V} = \phi$ and so $U \cap V = \phi$ .Then $U \subseteq V^c \subseteq K^c$. So $\overline{U} \subseteq \overline{V^c}$ $= V^c \subseteq K^c$. Writing U=H we have $x \in H \subseteq \overline{H} \subseteq K^c$.

Theorem 3.4:A topological space X is stronglypseudo-regular if X is completelyHausdorff .

Proof: Let X be a completely Hausdorff space and K be a compact subset of X. Let x, y be two distinct points of X with $y \in K$ and $x \notin K$ .Since X is completelyHausdorff there exist open sets G_y and H_y such that $x \in G_y$ and $y \in H_y$ and $\overline{G_y} \cap \overline{H_y} = \phi$. Let $\{H_y : y \in K\}$ is a open cover of K.

Since K is compact, so there exist a finite subcover $\{H_{y_1}, H_{y_2}, \ldots, H_{y_n}\}$of K. Let $H = H_{y_1} \cup H_{y_2} \cup \ldots \cup H_{y_n}$ and $G = G_{y_1} \cap G_{y_2} \cap \ldots \cap G_{y_n}$. Then $K \subseteq H$, $x \in G$ and we claim that $\overline{G} \cap \overline{H} = \phi$.

If $\overline{G} \cap \overline{H} \neq \phi$, let $z \in \overline{G} \Rightarrow z \in \overline{G_{y_1}} \cap \ldots \cap \overline{G_{y_n}}$ and $z \in \overline{H} \Rightarrow z \in \overline{H_{y_i}}$, for some y_i. This implies $z \in \overline{G_{y_i}} \cap \overline{H_{y_i}}$, which is a contradiction. Therefore $\overline{G} \cap \overline{H} = \phi$. Hence X is stronglypseudo-regular.

Theorem 3.5: The product space X of any non-empty collection $\{X_i\}$of topological spaces is stronglypseudo-regular if and only if each X_i is stronglypseudo-regular.

Proof: Let $\{X_i\}$ be a non-empty collection of stronglypseudo-regular space and X $=\Pi X_i$. We show that X is stronglypseudo-regular space. Let K be a compact set not containing a point $x \in X$. Let $K_i = \Pi_i(K)$, $x_i \notin K_i$. Since the projection maps are continuous $\Pi_i(K) = K_i$ is a compact subset of X_i. Since $x \notin K$, there exists i_0 such that $x_{i_0} \notin K_{i_0}$. Since X_{i_0} is stronglypseudo-regular,there exist open sets G_{i_0}, H_{i_0} in X_i such that $x_{i_0} \in H_{i_0}$, $K_{i_0} \subseteq G_{i_0}$ and $\overline{G_{i_0}} \cap \overline{H_{i_0}} = \phi$. For each $i \neq i_0$, let G_i, H_i be open sets such that $x_i \in H_i, K_i \subseteq G_i$. Let G=$\Pi_i G_i$ and H=$\Pi_i H_i$. Then $\overline{G} \cap \overline{H} = \phi$, since $\overline{G_{i_0}} \cap \overline{H_{i_0}} = \phi$ and $K \subseteq G$, $x \in H$. Hence X is strongly pseudo-regular.

Conversely, if X is stronglypseudo-regular, then we show thatfor each i , X_i is stronglypseudo-regular. For each i, let K_i be a compact subset of X_i and $x_i \in X_i$ but $x_i \notin K_i$. Let $K = \Pi_i K$ and x=$\{x_i\}$ then $x \in X$ but $x \notin K$. Then K compact by Tychonoff Theorem. Since X is stronglypseudo-regular, there exist open sets G and H such that $x \in G$ and $K \subseteq H$ and $\overline{G} \cap \overline{H} = \phi$ andG=$\Pi_i G_i$, H=$\Pi_i H_i$, G_i, H_i areopen sets in X_i such that $x_i \in H_i, K_i \subseteq G_i$ and $\overline{G_i} \cap \overline{H_i} = \phi$. Therefore X_i is stronglypseudo-regular.

Theorem 3.6: Let X be a stronglypseudo-regular space and R is an equivalence relation of X. Then R is a closed subset of X×X.

Proof: We shall prove that R^c is open. So, let $(x, y) \in R^c$. It is sufficient to show that there exist two open sets G and H of X such that $x \in G$ and $y \in H$ and G×H$\subseteq R^c$. Let p:X$\rightarrow \frac{X}{R}$ be the projection map. Since $(x, y) \in R^c$, $p(x) \neq p(y)$ i.e; $x \notin p^{-1}(p(y))$. Again, since $\{y\}$ is compact and p is a continuous mapping, p(y) is compact. Also, let $\{G_i\}$be an open cover of $p^{-1}(p(y))$ in X, and let $\overline{G_i} = p(G_i)$. Then $\{\overline{G_i}\}$ is an open cover of p(y) in $\frac{X}{R}$. Since p(y) is a singleton element in $\frac{X}{R}$, there exists $\overline{G_{i_0}}$ such that $p(y) \in \overline{G_{i_0}}$ in $\frac{X}{R}$. Then by the definition of the topology in $\frac{X}{R}$ and the nature of the map p, (i) G_{i_0} is open in X, (ii) $G_{i_0} = p^{-1}(\overline{G_{i_0}})$ and (iii) $p^{-1}(p(y)) \subseteq G_{i_0}$ in X. Hence $p^{-1}(p(y))$ is compact in X. So by the strongly pseudo-regularity of X there exist open sets G and H in X such that $x \in G$ and $p^{-1}(p(y)) \subseteq H$ and $\overline{G} \cap \overline{H} = \phi$. Hence $y \in p^{-1}(p(y)) \subseteq H$ i.e; $y \in H$. Since $\overline{G} \cap \overline{H} = \phi$, $p(\overline{G}) \cap p(\overline{H}) = \phi$. Therefore G×H$\subseteq R^c$ and so $(x, y) \in G \times H \subseteq R^c$.

Corollary 3.2: Let X be a stronglypseudo-regular space and R is an equivalence relation of X. Then $\frac{X}{R}$ is completely Hausdorff.

Proof: Let cls x and cls y be two distinct points of $\frac{X}{R}$. Then clsx= $p(x)$ and clsy= $p(y)$for some $x, y \in X$ such that $x \neq y$ and $(x, y) \in R^c$. By the proof of the above theorem, there exist open sets G_x and G_y in $\frac{X}{R}$ such that cls $x \in G_x$ and cls $y \in G_y$ and $\overline{G_x} \cap \overline{G_y} = \phi$. Thus $\frac{X}{R}$ is completelyHausdorff.

We shall now define a new class ofspecializedpseudo normal spaces (see [17]), viz.,strongly pseudo-normal spaces and proceed to study them.

IV. STRONGLY PSEUDO-NORMAL SPACES

Definition 4.1: A topological space X will be called**strongly pseudo-normal** if, for each pair of disjoint compact subsets K_1, K_2 of X, there existopen sets G and H such that $K_1 \subseteq G$, $K_2 \subseteq H$ and $\overline{G} \cap \overline{H} = \phi$.

Ex 4.1: X= ℝ with usual topology is strongly pseudo-normal. To see this, let K_1 and K_2 be two non-empty disjoint compact sets in X. Then, K_1 and K_2 may be written as $K_1=\bigcup_{i=1}^{\infty}[a_i,b_i]$, $K_2=\bigcup_{j=1}^{\infty}[c_j,d_j]$ where $[a_i,b_i]\cap[a_{i'},b_{i'}]=\phi$ if $i\neq i'$, $[c_j,d_j]\cap[c_{j'},d_{j'}]=\phi$ if $j\neq j'$ and $[a_i,b_i]\cap[c_j,d_j]=\phi$ for each i and j.

For each consecutive pair $[a_i,b_i]$ and $[c_j,d_j]$ in the natural ordering in ℝ , let

$$\partial_{ij}=\frac{1}{3}\inf\{|x-y|:x\in[a_i,b_i],y\in[c_j,d_j]\}$$

and let

$$V_{ij}=(a_i-\partial_{ij,}\,b_i+\partial_{ij,}),$$

$$W_{ij}=(c_j-\partial_{ij,}\,d_j+\partial_{ij,}).$$

Then, each V_{ij} and each W_{ij} are open, and

$V_{ij}\cap W_{ij}=\phi$. Let $V=\bigcup_{i,j}V_{ij}$ and $W=\bigcup_{i,j}W_{ij}$. Then, V and W are open, $\overline{V}\cap\overline{W}=(\bigcup_{i,j}\overline{V_{ij}})\cap(\bigcup_{i,j}\overline{W_{ij}})=\bigcup_{i,j}(\overline{V_{ij}}\cap\overline{W_{ij}})=\phi$ and $K_1\subseteq V$, $K_2\subseteq W$.

Thus, X is strongly pseudo-normal.

In the above $\overline{V}=\bigcup_{i,j}\overline{V_{ij}}$, $\overline{W}=\bigcup_{i,j}\overline{W_{ij}}$, because of the nature of V_{ij}'s and W_{ij}'s .

Theorem 4.1: Every strongly pseudo-normal space is pseudo normal but the converse is not true in general.

Proof: Let X be strongly pseudo-normal. Let K_1, K_2 be two disjoint compact subsets of X. Since X is strongly pseudo-normal, there exist open sets G and H such that $K_1\subseteq G$, $K_2\subseteq H$ and $\overline{G}\cap\overline{H}=\phi$.Since $\overline{G}\cap\overline{H}=\phi$, so $G\cap H=\phi$ Thus X is pseudo normal.

Conversely, let X={a,b,c,d} and $\Im=\{X,\phi,\{a\},\{b,c\},\{a,b,c\}\}$. Then $\Im$ is a topology on X. The closed subsets of X are $X,\phi,\{b,c,d\},\{a,d\},\{d\}$. Let $K_1=\{a\}$, $K_2=\{b\}$.Then $K_1$and K_2 are two disjoint compact subsets of X .We have open sets G= $\{a\}$, H= $\{b,c\}$such that $K_1\subseteq G$, $K_2\subseteq H$ and G$\cap$H$=\phi$. Hence X is pseudo normal. Clearly, G and H are the only disjoint open sets which separateK_1 and K_2 respectively.

We have $\overline{G}=\{b,c,d\}$, $\overline{H}=\{a,d\}$and so $\overline{G}\cap\overline{H}=\{d\}\neq\phi$. Hence X is not strongly pseudo-normal.

Theorem 4.2: Every open image of a stronglypseudo-normal space is stronglypseudo-normal.

Proof: Let X be a stronglypseudo-normal space and Y a topological space and let $f:X\to Y$ be an open and onto mapping. Let K_1 and K_2 be two disjoint compact subsets in Y. Since f is open, f^{-1} is continuous, $f^{-1}(K_1)$ and $f^{-1}(K_2)$ are compact in X. Since X is stronglypseudo-normal,there exist open subsets U and V of X such that $f^{-1}(K_1)\subset U$ and $f^{-1}(K_2)\subset V$ and $\overline{U}\cap\overline{V}=\phi$. Again, since f is open, f(U) and f(V) are open in Y and $K_1\subset ff^{-1}(K_1)\subset f(U)$, $K_2\subset ff^{-1}(K_2)\subset f(V)$. Now $f(\overline{U})\cap f(\overline{V})=\phi$. Since f is open , f is also closed. Therefore $f(\overline{U})$ is closed, hence $f(\overline{U})=\overline{f(\overline{U})}$. Since $f(U)\subseteq f(\overline{U})$, $\overline{f(U)}\subseteq\overline{f(\overline{U})}=f(\overline{U})$. Similarly $\overline{f(V)}\subseteq\overline{f(\overline{V})}=f(\overline{V})$. Therefore $\overline{f(U)}\cap\overline{f(V)}=\phi$. Hence Y is stronglypseudo-normal.

Corollary 4.1: Every quotient space of a stronglypseudo-normal space is stronglypseudo-normal.

Proof: Let X be a stronglypseudo-normal space and R is an equivalence relation on X. Since the projection map p:X$\to\frac{X}{R}$ is open and onto, the corollary then follows from the above theorem.

Although a subspace of a normal space need not be normal (see [1], p. 109), we have the following theorem:

Theorem 4.3: Every subspace of a stronglypseudo-normal space is stronglypseudo-normal.

Proof: Let X be a stronglypseudo-normal space and $Y \subseteq X$. Let K_1 and K_2 be two disjoint compact subsets in Y. Since K_1 and K_2 are compact in Y, these are compact in X too. Since X is stronglypseudo-normal, there exist open sets U and V such that $K_1 \subset U$ and $K_2 \subset V$ and $\overline{U} \cap \overline{V} = \phi$. Let G=$U \cap Y$ and H=$V \cap Y$. Then G and H are open sets in Y with property that $K_1 \subseteq G$ and $K_2 \subseteq H$ and $\overline{G} \cap \overline{H} = \phi$. Hence Y is stronglypseudo-normal.

Comment4.1: A continuous image of a stronglypseudo-regular (a stronglypseudo-normal) space need not be stronglypseudo-regular (stronglypseudo-normal).

For, if (X, T_1) is a stronglypseudo-regular (a stronglypseudo-normal) space and (X, I) a space with the indiscrete topology, then the identity map $1_x : (X, T_1) \to (X, I)$ is continuous and onto. But (X, I) is not stronglypseudo-regular (stronglypseudo-normal).

Theorem 4.4: A topological space X is stronglypseudo-normal if for each pair of disjoint compact sets K_1 and K_2, there exist open set U such that $K_1 \subseteq U \subseteq \overline{U} \subseteq K_2^{\ c}$.

Proof: Let X be a strongly pseudo-normal and K_1, K_2 be two compact subsets of X and $K_1 \cap K_2 = \phi$. Since X isstrongly pseudo-normal, there exist open sets U, V such that $K_1 \subseteq U,\ K_2 \subseteq V$ and $\overline{U} \cap \overline{V} = \phi$ and so $U \cap V = \phi$. Then $U \subseteq V^c \subseteq K_2^{\ c}$. So $\overline{U} \subseteq \overline{V^c} = V^c \subseteq K_2^{\ c}$. Hence we have $K_1 \subseteq U \subseteq \overline{U} \subseteq K_2^{\ c}$.

Theorem 4.5: A topological space X is stronglypseudo-normal if X is completelyHausdorff .

Proof: Let X be a completely Hausdorff space and A,B be two disjoint compact subsets of X. Let $x \in A$ and $y \in B$. Then $x \neq y$. Since X is completelyHausdorff, there exist open sets G_y and H_y such that $x \in G_y$ and $y \in G_y$ and $\overline{G_y} \cap \overline{H_y} = \phi$. Obviously $\{H_y : y \in B\}$ is an open cover of B.Since B is compact, so there exist finite subcover $H_{y_1}, H_{y_2}, \ldots\ldots, H_{y_m}$ of B. Let $H_x = H_{y_1} \cup H_{y_2} \cup \ldots\ldots \cup H_{y_m}$ and $G_x = G_{y_1} \cap G_{y_2} \cap \ldots\ldots \cap G_{y_m}$. Then $B \subseteq H_x,\ x \in G_x$ and $\overline{G_x} \cap \overline{H_x} = \phi$ i.e; X is strongly pseudo-regular. So for each $x \in A$ there exist two open sets G_x and H_x of X such that $x \in G_x$ and $B \subseteq H_x$ and $\overline{G_x} \cap \overline{H_x} = \phi$. Hence $\{G_x : x \in A\}$ is an open cover of A. Since A is compact, so there exist finite subcover $G_{x_1}, G_{x_2}, \ldots\ldots, G_{x_n}$ of this cover A. Let $G = G_{x_1} \cup G_{x_2} \cup \ldots\ldots \cup G_{x_n}$ and $H = H_{x_1} \cap H_{x_2} \cap \ldots\ldots \cap H_{x_n}$. Then G, H are open sets of X and $A \subseteq G,\ B \subseteq H$ and $\overline{G} \cap \overline{H} = \phi$. Hence X is stronglypseudo-normal.

Theorem 4.6: Every stronglypseudo-normal space is stronglypseudo-regular.

Proof: Let X be a stronglypseudo-normal space. Let K be a compact subset of X and let $x \in X$ such that $x \notin K$. Therefore $\{x\}$ and K are disjoint compact subsets of X. Since X is stronglypseudo-normal there exist open sets G and H in X such that $\{x\} \subseteq G$ and $K \subseteq H$ i.e; $x \in G$ and $K \subseteq H$ and $\overline{G} \cap \overline{H} = \phi$. Hence X is stronglypseudo-regular.

Theorem 4.7: Every normal T_1- space is strongly pseudo-regular.

Proof: Let X be normal and T_1. Then by Theorem 3.10 of ([1], p. 108), for each $x \in X$ and for each open set G with $x \in G_x$, there exists an open set H_x in X such that $x \in H_x \subseteq \overline{H_x} \subseteq G_x$ ….(1)

LetK be a compact subset of X and let $y \in X$ such that $y \notin K$. We note that X is Hausdorff, hencefor each $x \in K$, there exist open sets G_x and V_x such that $x \in G_x, y \in V_x$ and $G_x \cap V_x = \phi$. By (1),there exists an open set H_x in X such that $x \in H_x \subseteq \overline{H_x} \subseteq G_x$. Clearly ϰ=$\{H_x \mid x \in K\}$ and so G=$\{G_{x_1}, \ldots\ldots, G_{x_n}\}$ is open cover of K. K being compact. ϰ has a finite subcover, say, $\{H_{x_1}, \ldots\ldots, H_{x_n}\}$. Let G=$G_{x_1} \cup \ldots\ldots \cup G_{x_n}$ and V=$V_{x_1} \cap \ldots\ldots \cap V_{x_n}$. Then G and V are open sets in K and $G \cap V = \phi$. Also if H=$H_{x_1} \cup \ldots\ldots \cup H_{x_n}$, then H is open, H⊇K and $x \in V$ and $H \cap V = \phi$. Since $\overline{H_{x_1}}, \ldots\ldots, \overline{H_{x_n}} \subseteq G, \overline{H_{x_1}} \cup \ldots\ldots \cup \overline{H_{x_n}}$ is contained in G and is disjoint from V. $\overline{H_{x_1}} \cup \ldots\ldots \cup \overline{H_{x_n}}$ is a closed set containing $H_{x_1} \cup .... \cup H_{x_n}$ and so $\overline{H} \subseteq \overline{H_{x_1}} \cup .... \cup \overline{H_{x_n}}$. Hence $\overline{H} \cap V = \phi$. Now,there exists an open set W in X such that $x \in W \subseteq \overline{W} \subseteq V$. Then $\overline{H_{x_1} \cup ..... \cup H_{x_n}} \cap \overline{W} = \phi$ i.e, $\overline{H} \cap \overline{W} = \phi$. Therefore X is strongly pseudo-regular.

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