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Hysteresis and Vibrational Reasonance in Two Coupled-Van Der Pol Oscillators Driven by An Amplitude Modulated Signal

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Abstract— We consider two coupled Duffing-vander Pol (DVP) oscillators with a nonlinear coupling and driven by an amplitude modulated (AM) signal. We numerically study the effect of nonlinear coupling (δ), low and high-frequency components of AM signal for the cases of four types of potentials, namely, single-well, double-well, four-well and four-hump potentials. We show the occurrence of hysteresis and vibrational resonance (VR) for specific set of values of the parameters of the system. We characterize the phenomena such as hysteresis and VR by using bifurcation diagram and response amplitude (Q).

Keywords— Coupled DVP oscillators, Nonlinear coupling, AM signal, Hysteresis, Chaos, Vibrational resonance.

I. INTRODUCTION

The fundamental study of coupled nonlinear oscillators is significant in understanding the emergent behavior of complex dynamical systems such as bifurcation, multi-stability, hysteresis, nonlinear electronics, vibrational resonance etc. [1-5]. In the present work, we consider the two-coupled Duffing-van der Pol (DVP) oscillators driven by an amplitude modulated (AM) signal,

$$\ddot{x} - d\dot{x}(1-x^2) + 2Ax + 4\alpha x^3 + 2\delta x u^2 = (f + 2g\cos\Omega t)\sin\omega t \quad (1a)$$

$$\ddot{u} - d\dot{u}(1-u^2) + 2Bu + 4\beta u^3 + 2\delta u x^2 = 0 \quad (1b)$$

Where *d* is the coefficient of nonlinear damping, *A* and *B* are the natural frequencies, α and β are the coefficient of nonlinear terms, δ is the coupling parameter, *f* and *g* are the amplitudes of the driving force and ω and Ω are the frequencies of the driving force. In recent years, many studies were focused on the dynamics of two-coupled DVP oscillators [6-8]. Recently AM signal has been applied to certain nonlinear systems [9-11] to investigate certain nonlinear phenomena such as horseshoe chaos, hysteresis, VR etc. The choice d=0 and without AM signal give two-coupled anharmonic oscillator with the potential

$$V(x,u) = Ax^2 + Bu^2 + \alpha x^4 + \beta u^4 + \delta x^2 u^2 \quad (2)$$

The shape of the potential function (2) depends on the sign of the parameters. It can be a single-well, double-well, four-well and four-hump or unstable potential.

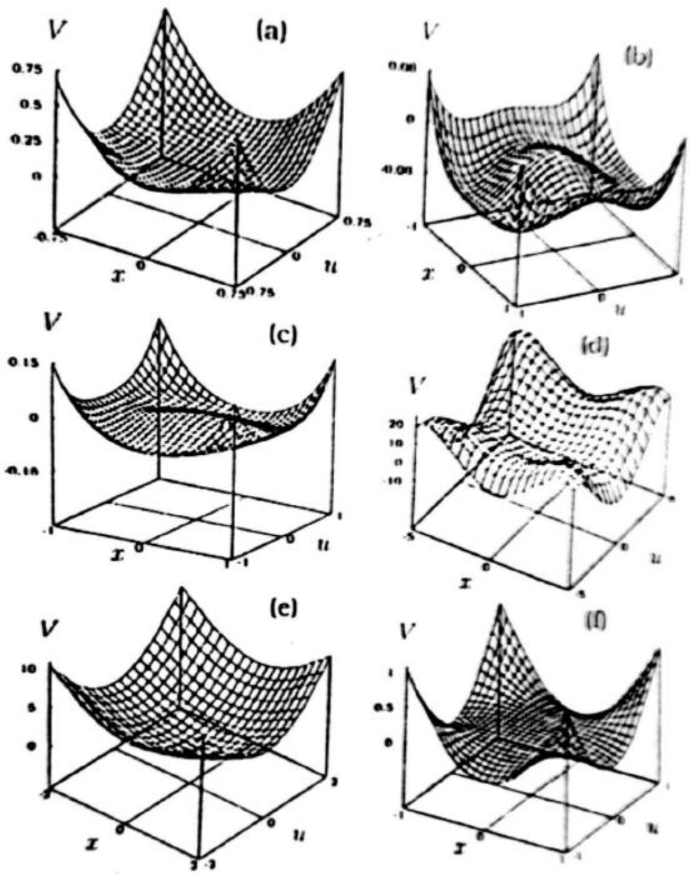

Fig.1: Different types of potentials V(x, u). (a) A single-well potential. The parameters are fixed as A = 0.1, B = 0.15, α = 0.1, β = 0.1 and δ = 0.15 A four-well potential. The parameters are fixed as A = -0.1, B = -0.15, α = 0.1, β = 0.1 and δ = 0.1.A double-well potential. The parameters are fixed as A = -0.1, B =- 0.15, α = 0.1, β = 0.1 and δ = 0.2. A four-hump potential. The parameters are fixed as A =1.0, B = 1.5, α = -0.06, β = -0.06 and δ = 0.05. A single-well potential. The parameters are fixed as A = 1.0, B = 1.5, α =-0.06, β =- 0.06 and δ = 0.15. An unstable potential. The parameters are fixed as A = -0.1, B = -0.15, α = -0.1, β =- 0.1 and δ = 1.5

II. HYSTERESIS AND VR WITH *A, B, A, B > 0*- SINGLE-WELL POTENTIAL

A large class of nonlinear dynamical systems is characterized by the coexistence of multiple attractors in some regions of the parameters space. The coexistence of several attractors give rise to the possibilities of hysteresis, that is, the possibilities of jumping through the coexisting of attractors in a way that is not reversible, when we fix a parameter back to its original value [12-14].

In a nonlinear dynamical system driven by a biharmonic force consisting of a low and high-frequencies ω and Ω with Ω >> ω. When the amplitude of the high frequency force is varied, the response amplitude at the low frequency ω exhibits a resonance**.** This high frequency induced resonance is called **Vibrational Resonance (VR)** [15-18]. To quantity the occurrence of VR, we use the response amplitude *Q* of the system (1) at the signal frequency ω. The system (1) is numerically integrated using fourth-order Runge-Kutta method with time step size (2π/ω)/1000. The first 10^3 drive cycles are left as transient. From the numerical solution of *x(t)*, the response amplitude *Q* is computed through $Q=\sqrt{(Q_s^2+Q_c^2)} / f$, where

$$Q_s = \frac{2}{nT}\int_0^{nT} x(t)\sin\omega t \; dt \qquad (3a)$$

$$Q_c = \frac{2}{nT}\int_0^{nT} x(t)\cos\omega t \; dt \qquad (3b)$$

Where $T=(2\pi/\omega)$ is the period of the response and *n* is taken as 500.

In this section we show the occurrence of hysteresis and VR in system (1) (eq.1) with $A,B,\ \alpha,\ \beta > 0$. For all values of parameters at $A=0.1,\ B=0.15,\ \alpha=0.1, \beta=0.1,\ d=0.2$ and integrated with fourth-order Runge-Kutta method with time step $\Delta t=0.00$. For a range of values of *δ, f* and *g* the system (1) (eq.1) shows the various nonlinear behaviors such as, the coexistence of several attractors, hysteresis and VR. First we consider the effect of low-frequency component of AM signal alone, that is $g=0$ and we fix $\omega=0.1,\ \Omega=5.0$ and $\delta=0.2.$ Hysteresis is observed in the presence of low-frequency component signal. The bifurcation behavior for $f\ \epsilon[1.0,2.0]$ where *f* is varied from 1.0 in the forward direction (Fig.2(a)) and when *f* is decreased from 2.0 [Fig.2(b)], a different bifurcation behavior is obtained, Hence the system exhibits the hysteresis phenomenon when *f* is varied smoothly from a small value to a larger one and then back to the smaller value. The hysteresis phenomenon is not observed in the presence of high-frequency component of the AM signal and it is observed when *δ* is varied from small value in the forward and reverse directions.

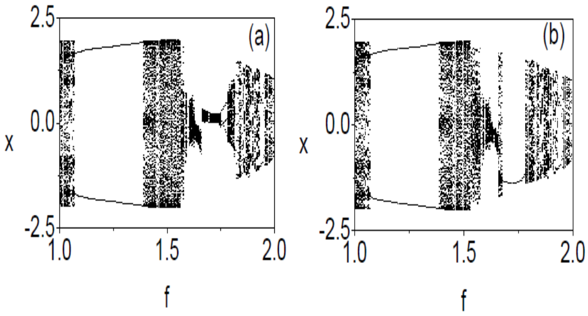

Fig.2: Bifurcation diagrams (a) *f* is varied in the forward direction from 1.0 (b) *f* is varied in the reverse direction from 2.0

In addition to the hysteresis, the system described by the eq.(1) exhibits the phenomenon of VR when the parameter *f, g* and *δ* are varied. Figure 3(a) shows the variation of numerically computed *Q* against the control parameter f for $\omega=0.1,\ \Omega=5.0,\ g=0.1$ and $\delta=0.2$. Multiple resonances occur, Maximum resonance peak occurs at $f\ =1.25$. Then we analyze the influence of the parameters *δ* and *g* on the resonance also. The results are presented in Figs.3(b) and 3(c). *Q* versus *g* is plotted in Fig.3(b) for $f=0.2, \delta=0.2$. Here again, multiple resonances occur for a range of *g* values. Maximum peak occurs at *g* = 1.985. The dependence of *Q* on the coupling parameter *δ* is presented in Fig.3(c) for *g* = 0.1 and *f* = 0.2. In the region $0 < \delta < 0.55$, no resonance occurs, when $\delta > 0.55$ multiple reosnances occur. This is clearly evident in Fig.3(c).

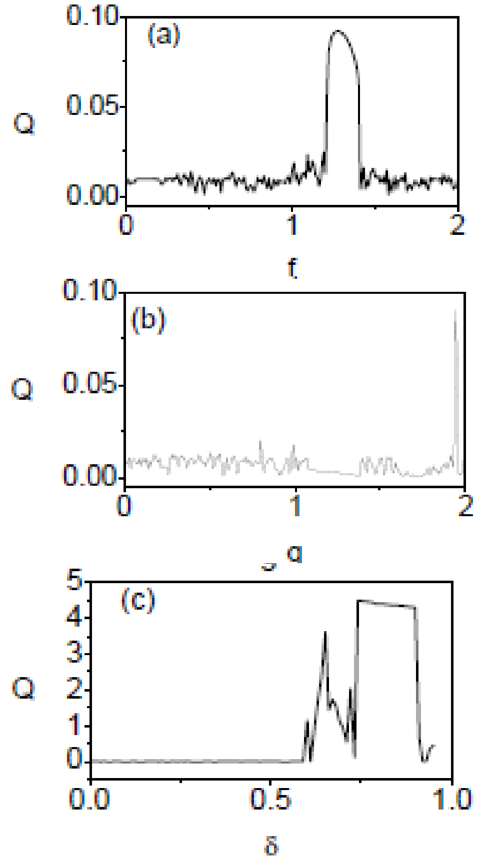

Fig.3: (a) Response amplitude *Q* versus *f* for $g = 0$ with $\delta = 0.2$ (b) *Q* versus *g* for $f = 0$ with $\delta = 0.2$. (c) *Q* versus *δ* with $f = 0.2$ and $g = 0.1$.

III HYSTERESIS AND VR WITH *A, B <0, A, B > 0* - DOUBLE-WELL OR FOUR WELL POTENTIALS

First we consider the system (1) (1) with $A,\ B\ <0,\ \alpha,\ \beta > 0.$ We fix the parameters at $A=-0.1,\ B=-0.15,\ \alpha=0.1, \beta=0.1, d=0.2$. For $0 < \delta < 2/15$, V is a four-well potential and is shown in Fig**.** (1b). For $2/15 < \delta < 0.3$, the potential is a double-well potential as shown in Fig.1(c). For $\delta > 0.3$, the potential has four-wells. That is, as *δ* increases from zero the shape of the potential changes from a four-well type to a double-well and then to another type of four-well. Coexistence of several attractors, hysteresis and VR are observed when the parameters *f, g* and *δ* are varied from zero for the four-well potential case. A bifurcation diagram plotted by varying *δ* in the forward direction as well as in the reverse direction is shown in Fig.4. Hysteresis and VR are obtained when *δ* is varied from small value. Similar behavior is also observed when *f* is varied in the forward and reverse directions from a small value. Hysteresis behavior is not observed due to the high-frequency component *(g)* of the AM signal. Now we analyze the VR behavior in system (1) (1) for the four-well potential case. Figure 5 shows the variation of numerically computed *Q* against the parameters *f, g* and *δ*. In all the parameters, we observed multiple resonances which are clearly evident in Figs.5(a), (b) and (c). In Fig.5(c), no resonance occurs for the region $0 < \delta < 0.56$. For $\delta > 0.56$, multiple resonances occur.

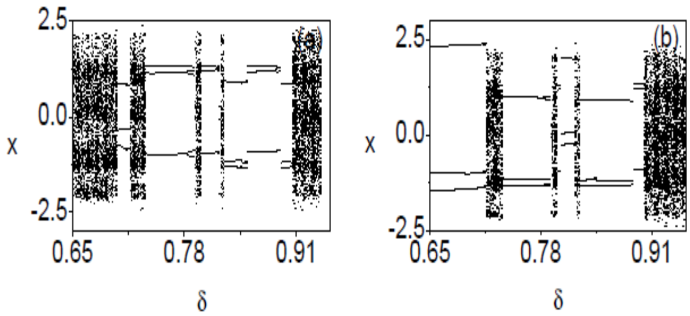

Fig.4: Bifurcation diagrams (a) δ is varied in the forward direction from 0.65 (b) δ is varied in the reverse direction from 0.95.

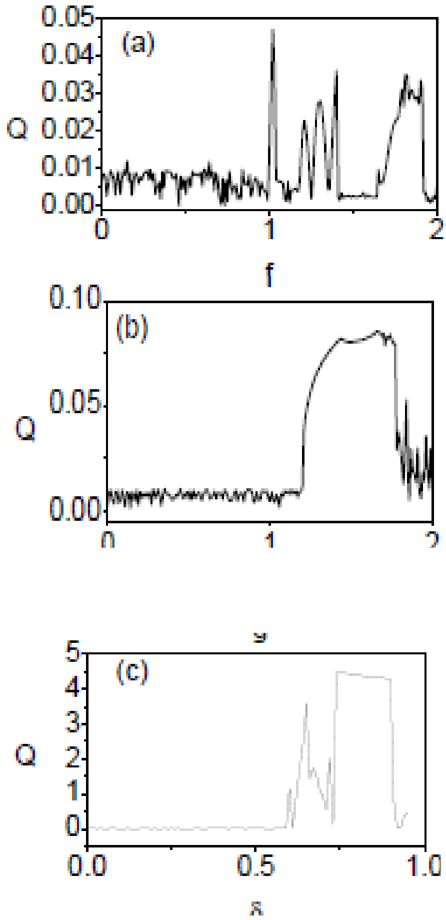

Fig.5: (a) Response amplitude Q versus f for $g = 0$ with $\delta = 0.2$ (b) Q versus g for $f = 0$ with $\delta = 0.2$. (c) Q versus δ with $f = 0.2$ and $g = 0.1$

IV HYSTERESIS AND VR WITH *A, B* > *0, A, B* < *0* - FOUR HUMP OR SINGLE-WELL POTENTIAL

We consider the system (1) (eq.1) with $A, B > 0$ and $\alpha, \beta < 0$. Now we fix the parameters as $A=1$, $B=1.5$, $\alpha=-0.06$, $\beta=-0.06$. For $o<\delta<0.12$ $(=\sqrt{4\alpha\beta})$, $V(x,u)$ is a four-hump potential (Fig.1(d)) and is a single-well potential [Fig(1e)] for $\delta > 0.12$. In this case, hysteresis phenomenon is not realized, but VR is observed for the different range of values of the parameters. Figure 6 illustrates the variation of numerically computed Q with the parameters f, g and δ. In Fig.6(a) when f is varied and $g=0$, $\delta=0.2$ multiple resonances are observed and maximum peak occurs at $f=0.38$. The variation of Q against g with $f=0$ and $\delta=0.2$ is shown in Fig.6 (b). Here again, we observed multiple resonances for all ranges of values of g. Finally we analyze the influence of the coupling strength (δ) on resonance. The results are presented in Fig.6(c). In this plot, multiple resonances occur for all ranges of value of δ. Maximum peak occurs at $\delta=0.2$, with $Q_{max}=0.03$ and $\delta=0.8$ with $Q_{max}=0.028$.

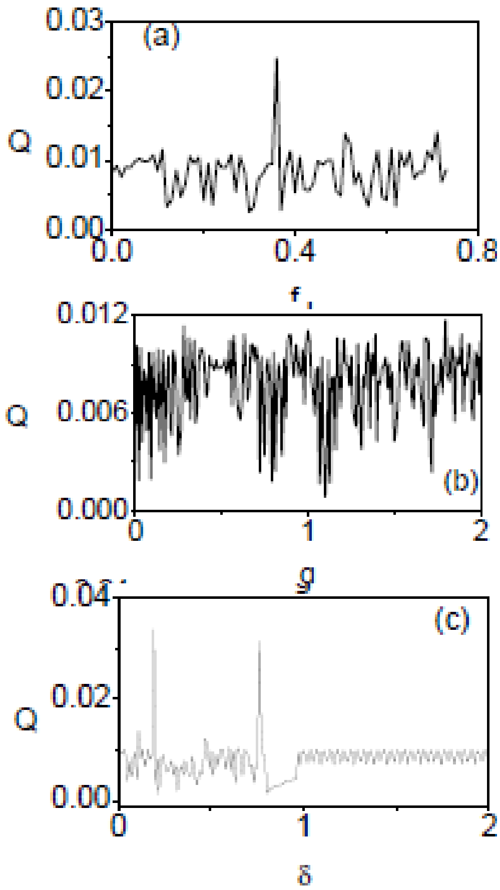

Fig.6: (a) Response amplitude Q versus f for $g = 0$ with $\delta = 0.2$ (b) Q versus g for $f = 0$ with $\delta = 0.2$. (c) Q versus δ with $f = 0.2$ and $g = 0.1$

CONCLUSION

We numerically studied the occurrence of hysteresis and vibrational resonance in system (1) driven by an AM signal for different shape of potentials, namely, single-well, double-well and four-well. The shape of the potential depends on the value of the parameter δ. For specific set of values of the control parameters f, g and δ, hysteresis phenomenon and VR are found for all the shape of the potentials. Apart from these phenomena, various types of bifurcations are also encountered in system (1). The occurrence of hysteresis and VR depend on the control parameters f, g and δ. From our numerical analysis, we observed multiple resonances for different values of the control parameters. It is of interest to investigate the occurrence of hysteresis and VR with system (1) with various types of coupling.

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