

Decomposition of rg^*b -Closed Sets in Supra Topological Spaces

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Abstract — In this paper, we introduce a new class of sets called supra rg^*b -locally closed sets and a new class of maps called supra rg^*b -locally continuous functions. Furthermore, we obtain some of their properties.

Keywords— S - rg^*b -LC sets, S - rg^*b -LC* sets, S - rg^*b -LC** sets, S - rg^*b -L-continuous and S - rg^*b -L-irresolute.

I. INTRODUCTION

Sindhu et al [8] defined and studied rg^*b -closed sets in Topological spaces. Bourbaki [1] defined a subset of space (X, τ) is called locally closed, if it is the intersection of an open set and a closed set. In topological space, some classes of sets namely generalized locally closed sets were introduced and investigated by Balachandran et al. [2]. Mashhour et al. [6] introduced the supra topological spaces and studied S -continuous functions and S^* -continuous functions. Indirani et al. [3] introduced and studied a class of sets and maps called supra rg^*b -closed sets and supra rg^*b -continuous maps.

In this paper we introduce the concept of supra rg^*b -locally closed sets and study its basic properties. Also we introduce the concepts of supra rg^*b -locally continuous maps and investigate some of its properties.

II. PRELIMINARIES

Throughout this paper, (X, τ) , (Y, σ) and (Z, η) (or simply, X , Y and Z) represent topological space on which no separation axioms are assumed, unless explicitly stated. For a subset A of (X, τ) , $cl(A)$ and $int(A)$ represent the closure of A with respect to τ and the interior of A with respect to τ , respectively. Let $P(X)$ be the power set of X . The complement of A is denoted by $X-A$ or A^c .

Now we recall some Definitions and results which are useful in the sequel.

Definition: 2.1 [6,7]

Let X be a non-empty set. The subfamily $\mu \subseteq P(X)$ is said to be a supra topology on X if $X \in \mu$ and μ is closed under arbitrary union. The pair (X, μ) is called a supra topological space.

The elements of μ are said to be supra open in (X, μ) . Complement of supra open sets are called supra closed sets.

Definition: 2.2 [7]

Let A be a subset of (X, μ) . Then

(i) The supra closure of a set A is, denoted by $cl^\mu(A)$, defined as $cl^\mu(A) = \bigcap \{B : B \text{ is a supra closed and } A \subseteq B\}$.

(ii) The supra interior of a set A is, denoted by $int^\mu(A)$, defined as $int^\mu(A) = \bigcup \{B : B \text{ is a supra open and } B \subseteq A\}$.

Definition: 2.3 [6]

Let (X, τ) be a topological space and μ be a supra topology of X . We call μ is a supra topology associated with τ if $\tau \subseteq \mu$.

Definition: 2.4 [4]

Let (X, τ) and (Y, σ) be two topological spaces and $\mu \subseteq \tau$. A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called supra continuous, if the inverse image of each open set of Y is a supra open set in X .

Definition: 2.5 [5]

Let (X, τ) and (Y, σ) be two topological spaces and μ and λ be supra topologies associated with τ and σ respectively. A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be supra irresolute, if $f^{-1}(A)$ is supra open set of X for every supra open set A in Y .

Definition: 2.6 [3]

A subset A of a supra topological space (X, μ) , is called Supra Regular Generalized star b -closed set (briefly rg^*b^μ -closed set) if $bcl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is rg^μ -open in X .

The family of all rg^*b -closed subsets of X is denoted by $RG^*B^\mu-C(X)$.

Definition: 2.7[3]

Let A be a subset (X, μ) . Then

- (i) The supra rg^*b -closure of a set A is, denoted by $rg^*b^\mu - cl(A)$, defined as $rg^*b^\mu - cl(A) = \bigcap \{B : B \text{ is supra } rg^*b\text{-closed and } A \subseteq B\}$
- (ii) The supra rg^*b -interior of a set A is, denoted by $rg^*b^\mu - int(A)$, defined as $rg^*b^\mu - int(A) = \bigcup \{B : B \text{ is supra } rg^*b\text{-open and } B \subseteq A\}$

III. SUPRA rg^*b -LOCALLY CLOSED SETS

In this section, we introduce the notion of supra rg^*b -locally closed sets and discuss some of their properties.

Definition: 3.1

Let (X, μ) be a supra topological space. A subset A of (X, μ) is called supra rg^*b -locally closed set (briefly supra rg^*b -LC set), if $A = U \cap V$, where U is supra rg^*b -open in (X, μ) and V is supra rg^*b -closed in (X, μ) .

The collection of all supra rg^*b -locally closed sets of X will be denoted by S - rg^*b -LC(X).

Remark: 3.2

Every supra rg^*b -closed set (resp. supra rg^*b -open set) is S - rg^*b -LC.

Definition: 3.3

For a subset A of a supra topological space (X, μ) , $A \in S$ - rg^*b -LC*(X, μ), if there exist a supra rg^*b -open set U and a

supra closed set V of (X, μ) , respectively such that $A=U \cap V$.

Definition: 3.4

For a subset A of (X, μ) , $A \in S\text{-}rg^*b\text{-}LC^{**}(X, \mu)$, if there exist a supra open set U and a supra $rg^*b\text{-}closed$ set V of (X, μ) , respectively such that $A=U \cap V$.

Definition: 3.5

Let (X, μ) be a supra topological space. If the space (X, μ) is called a supra $RG^*B\text{-space}$, then the collection of all supra $rg^*b\text{-open}$ subsets of (X, μ) is closed under finite intersection.

Definition: 3.6

Let $A, B \subseteq (X, \mu)$. Then A and B are said to be supra $\beta\text{-separated}$ if $A \cap rg^*b^{\mu} - cl(B) = B \cap rg^*b^{\mu} - cl(A) = \emptyset$.

Theorem: 3.7

Let A be a subset of (X, μ) . If $A \in S\text{-}rg^*b\text{-}LC^*(X, \mu)$ or $A \in S\text{-}rg^*b\text{-}LC^{**}(X, \mu)$, then A is $S\text{-}rg^*b\text{-}LC$.

Proof:

The proof is obvious by Definitions and the following example.

Example: 3.8

Let $X = \{a, b, c\}$ and $\mu = \{\emptyset, X, \{a, b\}, \{a, c\}\}$. Then $\{b, c\} \in S\text{-}rg^*b\text{-}LC$ but not $S\text{-}rg^*b\text{-}LC^*$ and $S\text{-}rg^*b\text{-}LC^{**}$.

Theorem: 3.9

For a subset A of (X, μ) , the following are equivalent:

- (i) $A \in S\text{-}rg^*b\text{-}LC^*(X, \mu)$.
- (ii) $A = U \cap cl^{\mu}(A)$, for some supra $rg^*b\text{-open}$ set U .
- (iii) $cl^{\mu}(A) - A$ is supra $rg^*b\text{-closed}$.
- (iv) $A \cup [X - cl^{\mu}(A)]$ is supra $rg^*b\text{-open}$.

Proof:

(i) \Rightarrow (ii): Given $A \in S\text{-}rg^*b\text{-}LC^*(X, \mu)$. Then there exist a supra $rg^*b\text{-open}$ subset U and a supra closed subset V such that $A=U \cap V$. Since $A \subset U$ and $A \subset cl^{\mu}(A)$, $A \subset U \cap cl^{\mu}(A)$. Conversely, $cl^{\mu}(A) \subset V$ and hence $A = U \cap V \supset U \cap cl^{\mu}(A)$. Therefore, $A = U \cap cl^{\mu}(A)$.

(ii) \Rightarrow (i): Let $A = U \cap cl^{\mu}(A)$, for some supra $rg^*b\text{-open}$ set U . Then, $cl^{\mu}(A)$ is supra closed and hence $A = U \cap cl^{\mu}(A) \in S\text{-}rg^*b\text{-}LC^*(X, \mu)$.

(ii) \Rightarrow (iii): Let $A = U \cap cl^{\mu}(A)$, for some supra $rg^*b\text{-open}$ set U . Then $A \in S\text{-}rg^*b\text{-}LC^*(X, \mu)$. This implies U is supra $rg^*b\text{-open}$ and $cl^{\mu}(A)$ is supra closed. Therefore, $cl^{\mu}(A) - A$ is supra $rg^*b\text{-closed}$.

(iii) \Rightarrow (ii): Let $U = X - [cl^{\mu}(A) - A]$. By (iii), U is supra $rg^*b\text{-open}$ in X . Then $A = U \cap cl^{\mu}(A)$ holds.

(iii) \Rightarrow (iv): Let $Q = cl^{\mu}(A) - A$ be supra $rg^*b\text{-closed}$. Then $X - Q = X - [cl^{\mu}(A) - A] = A \cup [X - cl^{\mu}(A)]$. Since $X - Q$ is supra $rg^*b\text{-open}$, $A \cup [X - cl^{\mu}(A)]$ is supra $rg^*b\text{-open}$.

(iv) \Rightarrow (iii): Let $U = A \cup [X - cl^{\mu}(A)]$. Since $X - U$ is supra $rg^*b\text{-closed}$ and $X - U = cl^{\mu}(A) - A$ is supra $rg^*b\text{-closed}$.

Theorem: 3.10:

For a subset A of (X, μ) , the following are equivalent:

- (i) $A \in S\text{-}rg^*b\text{-}LC(X, \mu)$.
- (ii) $A = U \cap rg^*b^{\mu} - cl(A)$, for some supra $rg^*b\text{-open}$ set U .
- (iii) $rg^*b^{\mu} - cl(A) - A$ is supra $rg^*b\text{-closed}$.
- (iv) $A \cup [X - rg^*b^{\mu} - cl(A)]$ is supra $rg^*b\text{-open}$.
- (v) $A \subseteq rg^*b^{\mu} - int(A \cup [X - rg^*b^{\mu} - cl(A)])$

Proof:

(i) \Rightarrow (ii): Given $A \in S\text{-}rg^*b\text{-}LC(X, \mu)$. Then there exist a supra $rg^*b\text{-open}$ subset U and a supra $rg^*b\text{-closed}$ subset V such that $A=U \cap V$. Since $A \subset U$ and $A \subset rg^*b^{\mu} - cl(A)$, $A \subset U \cap rg^*b^{\mu} - cl(A)$. Conversely, $rg^*b^{\mu} - cl(A) \subset V$ and hence $A = U \cap V \supset U \cap rg^*b^{\mu} - cl(A)$. Therefore, $A = U \cap rg^*b^{\mu} - cl(A)$.

(ii) \Rightarrow (i): Let $A = U \cap rg^*b^{\mu} - cl(A)$, for some supra $rg^*b\text{-open}$ set U . Then, $rg^*b^{\mu} - cl(A)$ is supra $rg^*b\text{-closed}$ and hence $A = U \cap rg^*b^{\mu} - cl(A) \in S\text{-}rg^*b\text{-}LC(X, \mu)$.

(ii) \Rightarrow (iii): Let $A = U \cap rg^*b^{\mu} - cl(A)$, for some supra $rg^*b\text{-open}$ set U . Then $A \in S\text{-}rg^*b\text{-}LC(X, \mu)$. This implies U is supra $rg^*b\text{-open}$ and $rg^*b^{\mu} - cl(A)$ is supra $rg^*b\text{-closed}$. Therefore, $rg^*b^{\mu} - cl(A) - A$ is supra $rg^*b\text{-closed}$.

(iii) \Rightarrow (ii): Let $U = X - [rg^*b^{\mu} - cl(A) - A]$. By (iii), U is supra $rg^*b\text{-open}$ in X . Then $A = U \cap rg^*b^{\mu} - cl(A)$ holds.

(iii) \Rightarrow (iv): Let $Q = rg^*b^{\mu} - cl(A) - A$ be supra $rg^*b\text{-closed}$. Then $X - Q = X - [rg^*b^{\mu} - cl(A) - A] = A \cup [X - rg^*b^{\mu} - cl(A)]$. Since $X - Q$ is supra $rg^*b\text{-open}$, $A \cup [X - rg^*b^{\mu} - cl(A)]$ is supra $rg^*b\text{-open}$.

(iv) \Rightarrow (iii): Let $U = A \cup [X - rg^*b^{\mu} - cl(A)]$. Since $X - U$ is supra $rg^*b\text{-closed}$ and $X - U = rg^*b^{\mu} - cl(A) - A$ is supra $rg^*b\text{-closed}$.

(iv) \Rightarrow (v): Since $A \cup [X - rg^*b^{\mu} - cl(A)]$ is supra $rg^*b\text{-open}$, $A \subseteq rg^*b^{\mu} - int(A \cup [X - rg^*b^{\mu} - cl(A)])$

(v) \Rightarrow (iv): Obvious.

Theorem: 3.12:

Let (X, μ) be a supra $RG^*B\text{-space}$ and $A \subset X$ be $S\text{-}rg^*b\text{-}LC$. Then

- (i) $rg^*b^{\mu} - int(A) \in S\text{-}rg^*b\text{-}LC(X, \mu)$.
- (ii) $rg^*b^{\mu} - cl(A)$ is contained in a supra $rg^*b\text{-closed}$ set
- (iii) A is supra $rg^*b\text{-open}$ if $rg^*b^{\mu} - cl(A)$ is supra $rg^*b\text{-open}$.

Proof:

(i) Let $A = U \cap rg^*b^{\mu} - cl(A)$, for some supra $rg^*b\text{-open}$ set U . Now, $rg^*b^{\mu} - int(A) = rg^*b^{\mu} - int(U \cap rg^*b^{\mu} - cl(A)) = rg^*b^{\mu} - int(U) \cap rg^*b^{\mu} - int(rg^*b^{\mu} - cl(A)) = rg^*b^{\mu} - int(U) \cap rg^*b^{\mu} - cl(rg^*b^{\mu} - int(A))$. Thus $rg^*b^{\mu} - int(A)$ is $S\text{-}rg^*b\text{-}LC(X, \mu)$.

(ii) $rg^*b^{\mu} - cl(A) = rg^*b^{\mu} - cl(U \cap rg^*b^{\mu} - cl(A)) \subset rg^*b^{\mu} - cl(U) \cap rg^*b^{\mu} - cl(A)$ which is a $rg^*b\text{-closed}$ set.

(iii) $rg^*b^{\mu} - int(A) = rg^*b^{\mu} - int(U \cap rg^*b^{\mu} - cl(A)) = rg^*b^{\mu} - int(U) \cap rg^*b^{\mu} - int(rg^*b^{\mu} - cl(A)) = U \cap rg^*b^{\mu} - cl(A) = A$, since $rg^*b^{\mu} - cl(A)$ is supra $rg^*b\text{-open}$. Hence A is supra $rg^*b\text{-open}$.

Theorem: 3.12:

If $A \subset B \subset X$ and B is $S\text{-}rg^*b\text{-}LC$, then there exists a $S\text{-}rg^*b\text{-}LC$ set C such that $A \subset C \subset B$.

Proof:

Immediate.

Theorem: 3.13:

For a subset A of (X, μ) , if $A \in S\text{-rg}^*b\text{-LC}^{**}(X, \mu)$, then there exist a supra open set G such that $A = G \cap cl^\mu(A)$

Proof:

Let $A \in S\text{-rg}^*b\text{-LC}^{**}(X, \mu)$. Then $A = G \cap V$, where G is a supra open set and V is a supra rg^*b -closed set. Then $A = G \cap V \Rightarrow A \subset G$. Obviously, $A \subset cl^\mu(A)$. Hence $A \subset G \cap cl^\mu(A)$ ---- (1). Also we have $cl^\mu(A) \subset V$. This implies $A = G \cap V \supset G \cap cl^\mu(A) \Rightarrow A \supset G \cap cl^\mu(A)$ ---- (2). From (1) and (2), we get $A = G \cap cl^\mu(A)$.

Theorem: 3.14:

For a subset A of (X, μ) , if $A \in S\text{-rg}^*b\text{-LC}^{**}(X, \mu)$, then there exist a supra open set G such that $A = G \cap rg^*b^\mu - cl(A)$.

Proof:

Let $A \in S\text{-rg}^*b\text{-LC}^{**}(X, \mu)$. Then $A = G \cap V$, where G is a supra open set and V is a supra rg^*b -closed set. Then $A = G \cap V \Rightarrow A \subset G$. Obviously, $A \subset rg^*b^\mu - cl(A)$. Hence $A \subset G \cap rg^*b^\mu - cl(A)$ ---- (1). Also we have $rg^*b^\mu - cl(A) \subset V$. This implies $A = G \cap V \supset G \cap rg^*b^\mu - cl(A) \Rightarrow A \supset G \cap rg^*b^\mu - cl(A)$ ---- (2). From (1) and (2), we get $A = G \cap rg^*b^\mu - cl(A)$.

Theorem: 3.15:

Let A be a subset of (X, μ) . If $A \in S\text{-rg}^*b\text{-LC}^{**}(X, \mu)$, then $rg^*b^\mu - cl(A) - A$ is supra rg^*b -closed and $A \cup [(X - rg^*b^\mu - cl(A))]$ is supra rg^*b -open.

Proof:

The proof is obvious from the Definitions and results.

Theorem: 3.16

Suppose (X, μ) is a supra RG^*B -space. Let $A \in S\text{-rg}^*b\text{-LC}^*(X, \mu)$ and $B \in S\text{-rg}^*b\text{-LC}^*(X, \mu)$. If A and B are supra rg^*b -separated, then $A \cup B \in S\text{-rg}^*b\text{-LC}(X, \mu)$.

Proof:

Let $A \in S\text{-rg}^*b\text{-LC}(X, \mu)$ and $B \in S\text{-rg}^*b\text{-LC}(X, \mu)$. By Theorem: 3.9, there exist supra rg^*b -open sets P and S of (X, μ) such that $A = P \cap cl^\mu(A)$ and $B = S \cap cl^\mu(B)$. Put $L = P \cap [X - cl^\mu(B)]$ and $M = S \cap [X - cl^\mu(A)]$. Then $L \cap rg^*b^\mu - cl(A) = [P \cap (X - rg^*b^\mu - cl(B))] \cap rg^*b^\mu - cl(A) = P \cap (rg^*b^\mu - cl(B))^c \cap rg^*b^\mu - cl(A) = A \cap (rg^*b^\mu - cl(B))^c = A$, since $A \subset (rg^*b^\mu - cl(B))^c$. Similarly, $M \cap rg^*b^\mu - cl(B) = B$. Then $L \cap rg^*b^\mu - cl(B) = \emptyset$ and $M \cap rg^*b^\mu - cl(A) = \emptyset$. Since X is a supra RG^*B -space, L and M are supra rg^*b -open. $(L \cup M) \cap L \cap rg^*b^\mu - cl(A \cup B) = (L \cup M) \cap (rg^*b^\mu - cl(A) \cup rg^*b^\mu - cl(B)) = (L \cap rg^*b^\mu - cl(A)) \cup (L \cap rg^*b^\mu - cl(B)) \cup (M \cap rg^*b^\mu - cl(A)) \cup (M \cap rg^*b^\mu - cl(B)) = A \cup B$. Therefore $A \cup B \in S\text{-rg}^*b\text{-LC}(X, \mu)$.

Definition: 3.17

Let (X, μ) be a supra topological space. A subset A of (X, μ) is called supra rg^*b -dense, if $rg^*b^\mu - cl(A) = X$.

Definition: 3.18

A supra topological space (X, μ) is called supra rg^*b -submaximal, if every supra rg^*b -dense subset is supra rg^*b -open in X .

Theorem: 3.19

A supra topological space (X, μ) is supra rg^*b -submaximal if and only if $P(X) = S\text{-rg}^*b\text{-LC}(X)$ holds.

Proof:

Necessity: Let $A \in P(X)$ and $G = A \cup [X - rg^*b^\mu - cl(A)]$. Then $rg^*b^\mu - cl(G) = X$ and so G is supra rg^*b -dense and hence supra rg^*b -open by assumption. By Theorem: 3.10, $A \in S\text{-rg}^*b\text{-LC}(X)$. Hence $P(X) = S\text{-rg}^*b\text{-LC}(X)$.

Sufficiency: Let every subset of X be supra rg^*b -locally closed. Let A be supra rg^*b -dense in X . Then $rg^*b^\mu - cl(A) = X$. Now $A = A \cup [X - rg^*b^\mu - cl(A)]$. By Theorem: 3.10, A is supra rg^*b -open. Hence X is supra rg^*b -submaximal

Theorem: 3.20

Let (X, μ) and (Y, λ) be the supra topological spaces.

- (1) If $M \in S\text{-rg}^*b\text{-LC}(X, \mu)$ and $N \in S\text{-rg}^*b\text{-LC}(Y, \lambda)$, then $M \times N \in S\text{-rg}^*b\text{-LC}(X \times Y, \mu \times \lambda)$.
- (2) If $M \in S\text{-rg}^*b\text{-LC}^*(X, \mu)$ and $N \in S\text{-rg}^*b\text{-LC}^*(Y, \lambda)$, then $M \times N \in S\text{-rg}^*b\text{-LC}^*(X \times Y, \mu \times \lambda)$.
- (3) If $M \in S\text{-rg}^*b\text{-LC}^{**}(X, \mu)$ and $N \in S\text{-rg}^*b\text{-LC}^{**}(Y, \lambda)$, then $M \times N \in S\text{-rg}^*b\text{-LC}^{**}(X \times Y, \mu \times \lambda)$.

Proof:

Let $M \in S\text{-rg}^*b\text{-LC}(X, \mu)$ and $N \in S\text{-rg}^*b\text{-LC}(Y, \lambda)$. Then there exist a supra rg^*b -open sets P and P' of (X, μ) and (Y, λ) and supra rg^*b -closed sets Q and Q' of (X, μ) and (Y, λ) respectively such that $M = P \cap Q$ and $N = P' \cap Q'$. Then $M \times N = (P \times P') \cap (Q \times Q')$ holds. Hence $M \times N \in S\text{-rg}^*b\text{-LC}(X \times Y, \mu \times \lambda)$. Similarly, the proofs of (2) and (3) follow from the Definitions.

IV. SUPRA rg^*b LOCALLY CONTINUOUS FUNCTIONS

In this section we define a new type of functions called Supra rg^*b -locally continuous functions ($S\text{-rg}^*b\text{-L}$ -continuous functions), supra rg^*b -locally irresolute functions and study some of their properties.

Definition: 4.1

Let (X, τ) and (Y, σ) be two topological spaces and $\tau \subseteq \mu$. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called $S\text{-rg}^*b\text{-L}$ -continuous (resp., $S\text{-rg}^*b\text{-L}^*$ -continuous, resp., $S\text{-rg}^*b\text{-L}^{**}$ -continuous), if $f^{-1}(A) \in S\text{-rg}^*b\text{-LC}(X, \mu)$ (resp., $f^{-1}(A) \in S\text{-rg}^*b\text{-LC}^*(X, \mu)$, resp., $f^{-1}(A) \in S\text{-rg}^*b\text{-LC}^{**}(X, \mu)$) for each $A \in \sigma$.

Definition: 4.2

Let (X, τ) and (Y, σ) be two topological spaces and μ and λ be supra topologies associated with τ and σ respectively. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be $S\text{-rg}^*b\text{-L}$ -irresolute (resp., $S\text{-rg}^*b\text{-L}^*$ -irresolute, resp., $S\text{-rg}^*b\text{-L}^{**}$ -irresolute) if $f^{-1}(A) \in S\text{-rg}^*b\text{-LC}(X, \mu)$ (resp., $f^{-1}(A) \in S\text{-rg}^*b\text{-LC}^*(X, \mu)$, resp., $f^{-1}(A) \in S\text{-rg}^*b\text{-LC}^{**}(X, \mu)$) for each $A \in S\text{-rg}^*b\text{-LC}(Y, \sigma)$ (resp., $A \in S\text{-rg}^*b\text{-LC}^*(Y, \sigma)$, resp., $A \in S\text{-rg}^*b\text{-LC}^{**}(Y, \sigma)$).

Theorem: 4.3

Let (X, τ) and (Y, σ) be two topological spaces and μ be a supra topology associated with τ . Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a function. If f is $S\text{-rg}^*b\text{-L}^*$ -continuous or $S\text{-rg}^*b\text{-L}^{**}$ -continuous, then it is $S\text{-rg}^*b\text{-L}$ -continuous.

Proof:

The proof is trivial from the Definitions.

Theorem: 4.4

Let (X, τ) and (Y, σ) be two topological spaces and μ and λ be supra topologies associated with τ and σ respectively. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function. If f is said to be $S\text{-}rg^*b\text{-}L$ -irresolute (resp., $S\text{-}rg^*b\text{-}L^*$ -irresolute, resp., $S\text{-}rg^*b\text{-}L^{**}$ -irresolute), then it is $S\text{-}rg^*b\text{-}L$ -continuous (resp., $S\text{-}rg^*b\text{-}L^*$ -continuous, resp., $S\text{-}rg^*b\text{-}L^{**}$ -continuous).

Proof:

The proof is trivial from the Definitions.

Theorem: 4.5

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be supra $rg^*b\text{-}LC$ -continuous and A be supra rg^*b -closed in X . Then the restriction $f|_A: A \rightarrow Y$ is $S\text{-}rg^*b\text{-}L$ -continuous.

Proof:

Let U be supra open in Y . Then $f^{-1}(U)$ in supra $rg^*b\text{-}LC$ in X . So $f^{-1}(U) = G \cap F$ where G is supra rg^*b -open and F is supra rg^*b -closed in X . Now $(f|_A)^{-1}(U) = (G \cap F) \cap A = G \cap (F \cap A)$ (resp. $(G \cap A) \cap F$) where $F \cap A$ is supra rg^*b -closed (resp. $G \cap A$ is supra rg^*b -open) in X . Therefore $(f|_A)^{-1}(U)$ is supra $rg^*b\text{-}LC$ in X . Hence $f|_A$ is supra $rg^*b\text{-}L$ -continuous.

Theorem: 4.6

A space (X, μ) is supra rg^*b -submaximal if and only if every function having (X, μ) as domain is supra $rg^*b\text{-}L$ -continuous.

Proof:

Necessity: Let (X, μ) be supra rg^*b -submaximal. Then $rg^*b\text{-}LC(X) = P(X)$ by Theorem: 3.19. Let $f: (X, \mu) \rightarrow (Y, \sigma)$ be a function and $A \in \sigma$. Then $f^{-1}(A) \in S\text{-}rg^*b\text{-}LC(X)$ and so f is $S\text{-}rg^*b\text{-}L$ -continuous.

Sufficiency: Let every function having (X, μ) as domain be supra $rg^*b\text{-}L$ -continuous. Let $Y = \{0, 1\}$ and $\sigma = \{\emptyset, Y, \{0\}\}$. Let $A \subset (X, \mu)$ and $f: (X, \mu) \rightarrow (Y, \sigma)$ be defined by $f(x) = 0$ if $x \in A$ and $f(x) = 1$ if $x \notin A$. Since f is supra $rg^*b\text{-}L$ -continuous, $A \in S\text{-}rg^*b\text{-}LC(X, \mu)$. Therefore $P(X) = S\text{-}rg^*b\text{-}LC(X)$ and so X is supra rg^*b -submaximal by Theorem: 3.19.

Theorem: 4.7

If $g: X \rightarrow Y$ is $S\text{-}rg^*b\text{-}L$ -continuous and $h: Y \rightarrow Z$ is supra continuous, then $hog: X \rightarrow Z$ is $S\text{-}rg^*b\text{-}L$ -continuous.

Proof:

Let $g: X \rightarrow Y$ is $S\text{-}rg^*b\text{-}L$ -continuous and $h: Y \rightarrow Z$ is supra continuous. By the Definitions, $g^{-1}(V) \in S\text{-}rg^*b\text{-}LC(X)$, $V \in Y$ and $h^{-1}(W) \in Y$, $W \in Z$. Let $W \in Z$. Then $(hog)^{-1}(W) = (g^{-1}h^{-1})(W) = g^{-1}(h^{-1}(W)) = g^{-1}(V)$, for $V \in Y$. From this, $(hog)^{-1}(W) = g^{-1}(V) \in S\text{-}rg^*b\text{-}LC(X)$, $W \in Z$. Therefore, hog is $S\text{-}rg^*b\text{-}L$ -continuous.

Theorem: 4.8

If $g: X \rightarrow Y$ is $S\text{-}rg^*b\text{-}L$ -irresolute and $h: Y \rightarrow Z$ is $S\text{-}rg^*b\text{-}L$ -continuous, then $hog: X \rightarrow Z$ is $S\text{-}rg^*b\text{-}L$ -continuous.

Proof:

Let $g: X \rightarrow Y$ is $S\text{-}rg^*b\text{-}L$ -irresolute and $h: Y \rightarrow Z$ is $S\text{-}rg^*b\text{-}L$ -continuous. By the Definitions, $g^{-1}(V) \in S\text{-}rg^*b\text{-}LC(X)$, for $V \in S\text{-}rg^*b\text{-}LC(Y)$ and $h^{-1}(W) \in S\text{-}rg^*b\text{-}LC(Y)$, for $W \in Z$. Let $W \in Z$. Then $(hog)^{-1}(W) = (g^{-1}h^{-1})(W) = g^{-1}(h^{-1}(W)) = g^{-1}(V)$, for $V \in S\text{-}rg^*b\text{-}LC(Y)$. This implies, $(hog)^{-1}(W) = g^{-1}(V) \in S\text{-}rg^*b\text{-}LC(X)$, $W \in Z$. Hence hog is $S\text{-}rg^*b\text{-}L$ -continuous.

Theorem: 4.9:

If $g: X \rightarrow Y$ and $h: Y \rightarrow Z$ are $S\text{-}rg^*b\text{-}L$ -irresolute, then $hog: X \rightarrow Z$ is also $S\text{-}rg^*b\text{-}L$ -irresolute.

Proof:

By the hypothesis and the Definitions, we have $g^{-1}(V) \in S\text{-}rg^*b\text{-}LC(X)$, for $V \in S\text{-}rg^*b\text{-}LC(Y)$ and $h^{-1}(W) \in S\text{-}rg^*b\text{-}LC(Y)$, for $W \in S\text{-}rg^*b\text{-}LC(Z)$. Let $W \in S\text{-}rg^*b\text{-}LC(Z)$. Then $(hog)^{-1}(W) = (g^{-1}h^{-1})(W) = g^{-1}(h^{-1}(W)) = g^{-1}(V)$, for $V \in S\text{-}rg^*b\text{-}LC(Y)$. Therefore, $(hog)^{-1}(W) = g^{-1}(V) \in S\text{-}rg^*b\text{-}LC(X)$, $W \in S\text{-}rg^*b\text{-}LC(Z)$. Thus hog is $S\text{-}rg^*b\text{-}L$ -irresolute.

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