COMBINATIONAL PDE BASED SPECKLE REDUCTION IN PYRAMID DOMAIN

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***Abstract*—A signal dependent noise called speckle is an inherent property of medical ultrasound (US) imaging modality, satellite aperture radar imaging (SAR) and optical coherence tomography (OCT) imaging. This speckle is multiplicative in nature and degrades the resolution, speed and accuracy of all the post processing tasks on the US/SAR/OCT imaging modalities. In this paper, a novel method has been proposed to reduce the speckle by applying a new combinational PDE which exhibits the properties of both second and fourth order partial differential equations (PDEs). The new PDE is applied in Laplacian pyramid domain to achieve better speckle reduction with edge preservation and feature enhancement.**

Keywords*—Diffusion; speckle; pyramid; image enhancement;

I. INTRODUCTION

The usefulness of US/SAR/OCT imaging are degraded by a signal dependent noise called speckle which is multiplicative in nature. Speckle is a granular pattern formed due to constructive and destructive coherent interferences of backscattered echoes from the scatterers that are typically much smaller than the spatial resolution (i.e., wavelength of an ultrasound wave) of medical US system [1].In medical diagnosis, the presence of speckle reduces the human interpretation ability and may leads to wrong diagnosis. So speckle noise reduction is an essential pre-processing step, and should be filtered out [2-5], without affecting important features of the image.

II. BACKGROUND

A. *PDE in Image Processing*

PDEs usage in image processing has grown considerably over the past two to three decades. The basic idea behind this frame work is to deform an image, a curve or a surface and to approach the expected result as a solution to this equation. Image denoising methods based on PDE utilize the physical process of diffusion to smooth noisy images. Image denoising methods utilize the physical process of diffusion [6] to smooth noisy images. Diffusion is generally defined as a physical process that equilibrates concentration differences without creating or destroying mass. By using Ficks law [6],

$$j = -D.\nabla I \tag{1}$$

Here, ∇I is concentration gradient that causes a flux j. This gradient ∇I and j are related by the diffusion tensor D,

B. *Second and Fourth order PDEs*

Since 1990, many researchers have studied second order PDE based methods for speckle reduction [3-8] , that is commonly given by,

$$\begin{cases} \dfrac{\partial I}{\partial t} = \nabla.[c(|\nabla I|)\nabla I] \\ I(t=0) = I_0 \end{cases} \tag{2}$$

Where ∇, $\nabla.$ are the gradient and the divergence operators, $\|$ denotes the magnitude, $c(x)$ is the diffusion coefficient and I_0 is the initial image. Many choices are available for $c(x)$ in the literature [1-8]. But using (2) for smoothing an image brings in "blocky effects" [4].

To avoid these blocky effects and to achieve good trade-off between noise removal and edge preservation, Fourth order PDE has been proposed [9-11] as,

$$\frac{\partial I}{\partial t} = -\nabla^2 [\, g(|\nabla^2 I|)\nabla^2 I \,] \tag{3}$$

Fourth-order PDE in image denoising removes the blocky effects that made by second-order nonlinear diffusion equation; however, it requires more number of iterations to converge and leaves the processed images with isolated black and white speckles.

C. *Laplacian Pyramid*

The image pyramid is a data structure designed to support efficient scaled convolution through reduced image representation. A general structure of pyramid transform consists of decomposition and reconstruction stages and can be described by approximation and interpolation filtering. The basic classifications of pyramids are: Gaussian pyramid and Laplacian Pyramid. . A specific pyramid is determined by its particular decimation factor, approximation and interpolation filters. In the Laplacian pyramid, two operators such as REDUCE and EXPAND are commonly used. The REDUCE operator performs a two-dimensional (2-D) low pass filtering followed by a sub-sampling by a factor of two in both directions. The EXPAND operator enlarges an image to twice the size in both directions by up-sampling (i.e., insertion of zeros) and a low pass filtering [1]. This filtering is followed by a multiplication by a factor of four, which is necessary to maintain the average intensity being reduced by the insertion of zeros.

The Gaussian (G) and Laplacian (L) pyramids are defined as:

$$\begin{aligned} G_0 &= I \\ G_l &= REDUCE[G_{l-1}] \\ L_l &= G_l - EXPAND[G_{l+1}] \end{aligned} \tag{4}$$

Reconstruction of an image from its Laplacian pyramid can be achieved by simply reversing the decomposition steps[12].

III. DEVELOPMENT OF COMBINATIONAL PDE

The Fick's law in (1) can be approximated as a super flux with near equilibrium as,

$$j(x,t) = -\sum_q D_q \nabla\nabla^{2q} I(x,t) \tag{5}$$

For an image system it can be expressed as,

$$\frac{\partial I(x,y,t)}{\partial t} = -\nabla.[D_1 \nabla I(x,y,t) + D_2 \nabla I(x,y,t)\nabla^2 I(x,y,t)] \tag{6}$$
$$if \quad D_1 = D_2 = c(|\nabla I|)$$

where ∇, Δ are gradient and Laplacian operators, $c(|\nabla I|)$ is diffusion coefficient. Thus, the combinational PDE consumes minimum number of iterations to converge and also predominantly reduces the blocky effects caused by second order anisotropic diffusion as it carries mixed order.

A. *Combinational PDE in Pyramid Domain*

Pyramid transform separates information into frequency bands. As speckle noise has high frequency, it mainly resides in fine scale corresponds to low pyramid layer and it is negligible in the coarser scale corresponds to the higher pyramid layer. Algorithm used in the pyramid domain is as follows;

Step 1: Decompose an image into its pyramid structure (3 layers) of decreasing frequencies

Step 2: Filter each band pass layer of Laplacian pyramid using combinational PDE

Step 3: Estimate gradient threshold in each layer '*l*' using median estimator [1],

$$k(l) = \frac{1}{0.6745} median\left(\frac{|\nabla I(l)|}{\sqrt{2\log((l+1)/l)}}\right)$$

Step 4: Use Mean absolute error (MAE) between two adjacent diffusion steps as a stopping criteria to stop the iterations,

$$MAE(I(t)) = \frac{1}{M \times N} \sum_{(i,j)=1}^{M,N} \sqrt{(I(i,j,t) - I(i,j,t-1))^2}$$

Step 5: Reconstruct the image from its Laplacian pyramid by simply reversing the decomposition steps

B. *Stability Criteria*

The convergence and stability of MAE [13] is verified by applying combinational PDE to three band pass layers and MAE value has been observed and it decreases exponentially with the number of iterations as shown in Fig.1.

C. *Numerical Aspects*

A finite difference scheme is preferred because of its easy implementation to solve the diffusion equation. The image gradients are obtained from directional differences using symmetric boundary conditions. The average of the four squared directional differences is used to discretize gradient and Laplacian operators.

$$\begin{aligned} |\nabla I| &= 0.5 \times \sqrt{|\nabla I_N|^2 + |\nabla I_S|^2 + |\nabla I_W|^2 + |\nabla I_E|^2} \\ &= 0.5 \times \sqrt{(I^n_{i-1,j} - I^n_{i,j})^2 + (I^n_{i+1,j} - I^n_{i,j})^2 + (I^n_{i,j-1} - I^n_{i,j})^2 + (I^n_{i,j+1} - I^n_{i,j})^2} \end{aligned}$$

$$\nabla^2 I^n_{i,j} = I^n_{i+1,j} + I^n_{i-1,j} + I^n_{i,j+1} + I^n_{i,j-1} - 4I^n_{i,j} \tag{7}$$

Thus the combinational PDE can be expressed as,

$$I^{n+1}_{i,j} = I^n_{i,j} + \lambda[c(|\nabla I^n_{i,j}|)\nabla I^n_{i,j} + c(|\nabla I^n_{i,j}|)\nabla I^n_{i,j}\nabla^2 I^n_{i,j}] \tag{8}$$

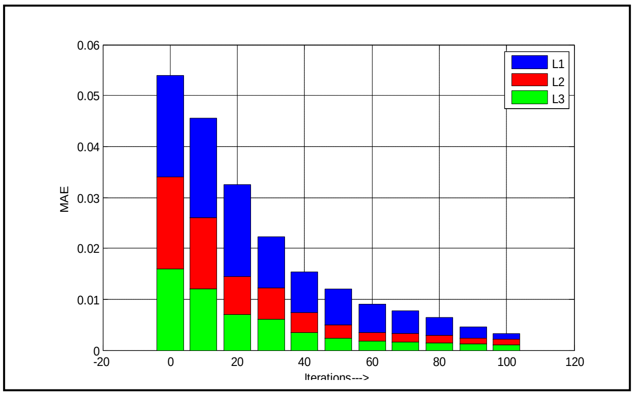


Fig.1. Mean Absolute Error – Stability and convergence

Where c is diffusion coefficient and λ is the time step that controls the swiftness of diffusion process. The Neumann boundary condition with an assumption that the values beyond an image border are equivalent to values on the border is utilized to solve the combinational PDE. The time step [14] is chosen as $\lambda \le 0.25$.

IV. RESULT AND CONCLUSION

Performance improvement of the proposed methods is measured through three metrics [8, 15,16]; the Mean squared error (MSE), Edge preservation index (EPI) and Structural similarity index (SSI).

TABLE I. PERFORMANCE METRICS

Measure	*Definition*	*Parameters*
MSE	$\frac{1}{M \times N}\sum_{(i,j)=1}^{M\times N}\left(\hat{S}(i,j)-S(i,j)\right)^2$	S - reference image $\hat{S}$ - filtered images
EPI	$\frac{\sum(\Delta S-\overline{\Delta S})(\Delta\hat{S}-\overline{\Delta\hat{S}})}{\sqrt{\sum(\Delta S-\overline{\Delta S})^2\sum(\Delta\hat{S}-\overline{\Delta\hat{S}})^2}}$	$S,\hat{S},\bar{S},\Delta S$ Are original, denoised image, mean of S and high pass filtered S using discrete Laplacian operator. Ci is the constant to avoid instability
SSI	$\frac{(2\bar{S}\bar{\hat{S}}+C_1)(2\sigma_{S\hat{S}}+C_2)}{(\bar{S}^2+\bar{\hat{S}}^2+C_1)(\sigma_S^2+\sigma_{\hat{S}}^2+C_2)}$	

The performance of the proposed CPDE method is evaluated using synthetic image and simulated phantom. The performance is compared with LPND [1], NLMS [17], FBLF [18] methods. The results are shown in Fig.2 and Fig.3.The performance measures are listed in Table.II.

TABLE II. PERFORMANCE COMPARISON

Methods	*Synthetic Image*			*Simulated Phantom*		
	MSE	***EPI***	***SSI***	***MSE***	***EPI***	***SSI***
LPND	56.89	0.682	0.736 ±0.003	58.22	0.712	0.768 ±0.003
NLMS	54.05	0.784	0.772 ±0.003	52.12	0.796	0.781 ±0.003
FBLF	43.45	0.798	0.794 ±0.029	44.67	0.822	0.787 ±0.003
CPDE	**28.81**	**0.899**	**0.828 ±0.002**	**22.23**	**0.931**	**0.8422 ±0.003**

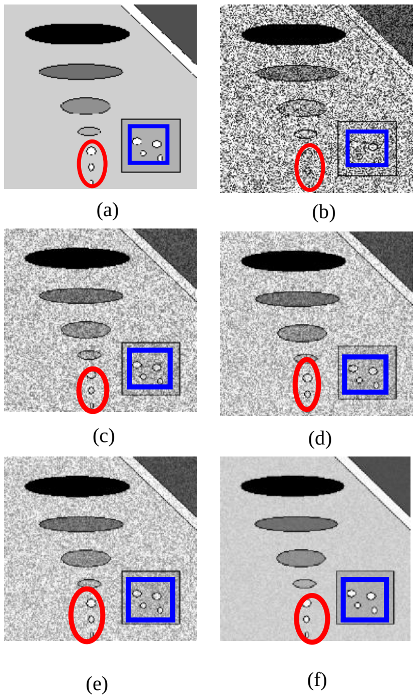


Fig.2. Result on synthetic image (a) Original (b) Noisy (c) LPND (d) NLMS (e) FBLF (f) Proposed CPDE

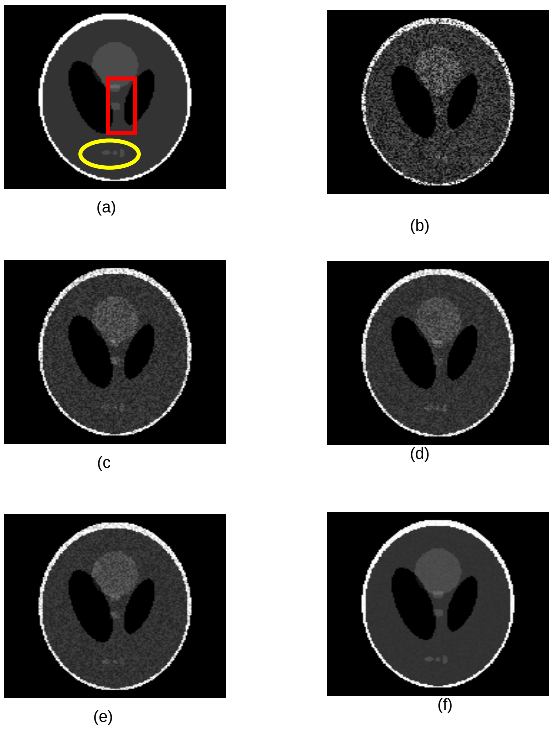
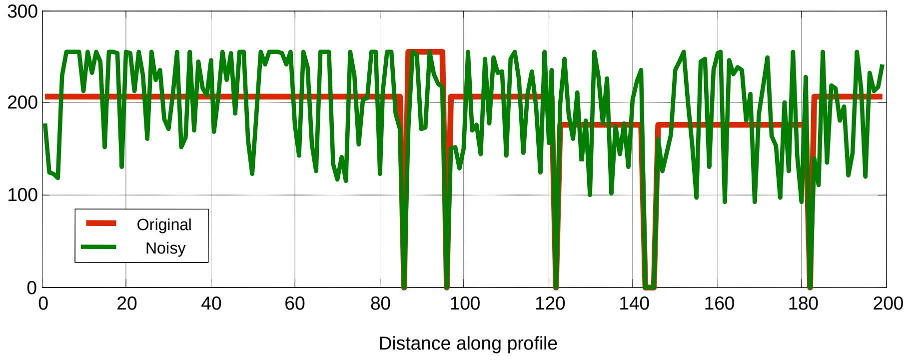
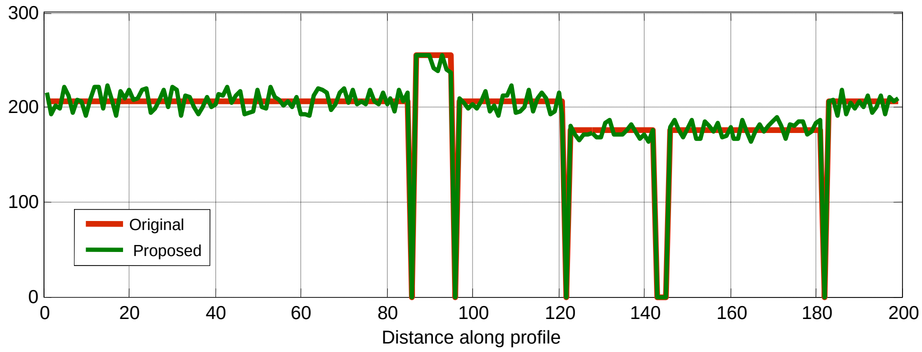


Fig.3. Result on Simulated Phantom (a) Original (b) Noisy (c) LPND (d) NLMS (e) FBLF (f) Proposed CPDE



(a)



(b)

Fig.4.Image profile along 100th row of simulated phntom (e) Noisy image (f) Proposed CPDE

In the comparative analysis, proposed CPDE provides maximum speckle reduction while preserving detailed structures. Enhancement is observed in the areas enclosed by red and blue in synthetic image, red and yellow in the simulated phantom. Fig.4 gives performance of proposed method in terms of image profile along 100th row in the simulated phantom and it clearly justifies the speckle reduction ability of proposed CPDE model.

Thus the proposed combinational PDE based diffusion model offers a tradeoff between second and fourth order PDE based methods in terms of avoiding blocky effects and number of iterations required to converge. This method can be further extended to support all kind of post processing methods on US/SAR/OCT imaging like registration, segmentation and feature extraction etc., and can also be used as a visual enhancement aid to justify the diagnosis in the medical field.

References

[1] Fan Zhang, Yang Mo Yoo, Liang Mong Koh, and Yongmin Kim,” Nonlinear Diffusion in Laplacian Pyramid Domain for Ultrasonic, Speckle Reduction” *IEEE Trans. Medicalimaging*, vol. 26, No. 2, Feb 2007.

[2] C.B. Burckhardt, “ Speckle in Ultrasound B Mode Scans,” IEEE Trans. Sonics Ultrasonics ,1978; pp.25:1-6.

[3] Zong X., Laine A. F., and Geiser E. A., “ Speckle reduction and contrast enhancement of echocardiograms via multiscale nonlinear processing,” IEEE Trans.on Medical. Imaging., 1998;17:532-540.

[4] Yongjian Y. and Acton S.T, “Speckle reducing anisotropic diffusion,” IEEE Trans. Image Processing. Nov. 2002; 11(11):1260–1270.

[5] Prager R.W., Gee A.H., Treece G.M., and Berman L.,” Speckle detection in ultrasound images using first order statistics,” GUED/ F-INFENG/TR 415, University of Cambridge, Dept. of Engineering, July, 2002: 1–17.

[6] Perona, P., and Malik, J., “ Scale-space and edge detection using anisotropic diffusion,”. IEEE Trans. Pattern Anal. and Machine Intell., vol. 12, Issue 7. (1990) 629-639

[7] Krissian, K., Westin, C. F., Kikinis, R. and Vosburgh, K. G., “Oriented speckle reducing anisotropic diffusion,”. IEEE Trans. Image Processing, vol. 16, Issue 5. (2007) 1412-1424

[8] Wang, B., Liu, D. C., “ Semi-Implicit Scheme based Nonlinear Diffusion Method in Ultra-sound Speckle Reduction,” IEEE International Ultrasonics Symposium, (2008) 1390-1393.

[9] Yu Li You and Kaveh, M., “Fourth-Order partial differential equations for Noise removal”. IEEE Trans. on Image processing, Vol.9, No.10, pp. 1723-1730, 2000.

[10] Lysaker, M., Lundervold, A. and Tai, X.C., “Noise removal using Fourth-Order partial differential equation with application to medical magnetic resonance images in space and time”, IEEE Trans. on Image processing, Vol.12, No.12, pp. 1579-1590, 2003

[11] I.S. Jacobs and C.P. Bean, "Fine particles, thin films and exchange anisotropy," in Magnetism, vol. III, G.T. Rado and H. Suhl, Eds. New York: Academic, 1963, pp. 271-350.

[12] P. J. Burt and E. A. Adelson, "The Laplacian pyramid **as** a compact image code,"*IEEE Trans. Contnnm.***,** vol. 31, pp. 532-540, Apr. 1983.

[13] Acton, S.T., "DeconVolutional speckle reducing anisotropic diffusion," IEEE International Conference on Image Processing, Vol.1, pp. 5-8, 11-14, 2005.

[14] Mrázek, J. Weickert, G. Steidl, and M. Welk, "On iterations and scales of Nonlinear filters," in Proc. Computer VisionWinterWorkshop, O. Drbohlav, Ed., Valtice, Czech Republic, pp. 61-66, 2003.

[15] Pratt, W. K., Digital Image Processing, Wiley, New York, NY, USA, 1978.

[16] Michailovich O, Tannenbaum, A, "Despeckling of ultrasound images", IEEE Trans. Ultrasounics Ferroelectrics and Frequency Control,
Vol. 53, No. 1 pp. 64-78, 2006

[17] P. Coupé, P. Hellier, C. Kervrann, and C. Barillot, "Nonlocal means-based speckle filtering for ultrasound images," *IEEE Trans. Image Process.*, vol. 18, pp. 2221-2229, 2009.

[18] K. N. Chaudhury, D. Sage, and M. Unser, "Fast bilateral filtering using trigonometric range kernels," *Image* Processing, IEEE Trans. Image Process., vol. 20, pp. 3376-3382, 2011