Reduction of Inter-Carrier-Interference Using Total Cancellation Method in OFDM System

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Abstract—In the area of wireless communications, the demand for high data rate transmission is rapidly increasing. Orthogonal frequency division multiplexing (OFDM) is known to be a promising technique for high-rate transmission that can overcome the inter symbol interference (ISI) which results from the time dispersive nature of wireless channels. For OFDM communication systems the orthogonality is lost among the sub-carriers due to frequency offset which results in Inter carrier Interference (ICI). This ICI rapidly degrades the performance of OFDM system. We have so many ICI cancellation methods like time windowing and frequency equalization to improve the BER performance of OFDM systems. In this an efficient ICI cancellation methods termed Kalman filter (KF) method and another ICI cancellation scheme, named Total ICI Cancellation scheme are proposed. However the total ICI cancellation scheme has does not lower the transmission rate or reduce the bandwidth efficiency. It is shown that for high values of the frequency offset and for higher order modulation schemes, the KF method perform better than other existing methods. The Total ICI Cancellation scheme takes advantage of the orthogonality of the ICI matrix and offers perfect ICI cancellation and significant BER improvement at linearly growing cost. Simulation results in AWGN channel confirm the superb performance of the Total ICI Cancellation scheme in the presence of frequency offset or time variations in the channel compared with other two schemes.

Keywords—Kalman Filter (KF), OFDM, ICI, AWGN, BER

I. INTRODUCTION

OFDM technique is basically a method of encoding digital data on multiple carrier frequencies. The basic principle of OFDM technique is to split the available spectrum into N number of sub-channel bandwidths and transmission of signal using orthogonal carriers through these sub-channels that is a large number of closely spaced orthogonal sub-carrier signals are used to carry a data on several parallel data streams or channels. OFDM converts the frequency selective channel to frequency flat channel so that it can completely eliminate Inter symbol interference (ISI). This is the major advantage of OFDM (which is multi carrier communication) over single carrier communication.

At its earlier version it was used as Single Input Single Output (SISO) system which used single antenna as Transmitter and single antenna as Receiver. This generated a limitation in the system that it was susceptible to fading effects in channel and if once information was corrupted or lost due to fading or noise interference it was needed to be retransmitted. This used to affect the spectral efficiency of the system so instead of single antenna system, multiple antennas were started being used as transmitter as well as receiver. The system was abbreviated as Multiple Input Multiple Output (MIMO) system.

The combination of MIMO with OFDM provides high data rate and spectral efficiency by providing transmission of parallel data streams over multiple antennas and it is being considered best suitable for 4G communication systems. It promises a significant boost in performance for OFDM systems. In MIMO OFDM systems there is a need of exact channel estimation at the receiver for the sake of invoking both coherent demodulation and interference cancellation. However performance of MIMO OFDM system is affected by Inter Symbol Interference (ISI), Inter Carrier Interference (ICI), and Peak to Average Power Ratio (PAPR).

Inter carrier interference is a result of frequency offset between Transmitter and Receiver. Due to this a loss of orthogonality among the subcarriers starts, so there is a power outflow between subcarriers which degrades performance of MIMO-OFDM system if not compensated properly.

II. MIMO-OFDM SYSTEM

Orthogonal Frequency Division Multiplexing (OFDM) is a multi-carrier modulation technique, in which a single high rate data-stream is divided into multiple low rate data streams and is modulated using sub-carriers which are orthogonal to each other. Major advantages of OFDM are its multi-path delay spread tolerance and efficient spectral usage by allowing overlapping in the frequency domain. Also another significant advantage is that the modulation and demodulation can be done using IFFT and FFT operations, which are computationally efficient. In addition to above, OFDM has several favorable properties like high spectral efficiency, robustness to channel fading, immunity to impulse interference, uniform average spectral density, capacity to handle very strong echoes and non-linear distortion.

In OFDM the signal itself is first split into independent channels, modulated by data and then re-multiplexed to create the OFDM carrier. The sub-carriers should be orthogonal to each other to improve spectral efficiency. It provides the main idea of OFDM based communication. At the receiver side data recovery in each sub-carriers is only possible when the orthogonality between these are maintained. As more and more carriers are added, the bandwidth approaches (N+1)/N Bits per Hz. Larger number of carriers gives better spectral efficiency. In OFDM the question of Multiplexing is applied to independent signals but these independent signals are a sub-set of the one main signal. The main concept in OFDM is Orthogonality of the sub-carriers.

The OFDM system transmits a large number of narrowband carriers, which are closely spaced. The Orthogonality among the
carriers can be maintained if the OFDM signal is defined by using Fourier transform procedures. Note that at the central frequency of the each sub channel there is no crosstalk from other sub channels. In an OFDM system, the input bit stream is multiplexed into N symbol streams, each with symbol period Ts, and each symbol stream is used to modulate parallel, synchronous sub-carriers. The sub-carriers are spaced by 1/NTs in frequency, thus they are orthogonal over the interval (0, Ts).

A typical discrete-time baseband OFDM transceiver system is shown in Figure 1. First, a serial-to-parallel (S/P) converter groups the stream of input bits from the source encoder into groups of \(\log_2 M\) bits, where M is the alphabet of size of the digital modulation scheme employed on each sub-carrier. A total of N such symbols, \(X_m\), are created. Then, the N symbols are mapped to bins of an inverse fast Fourier transform (IFFT).

These IFFT bins correspond to the orthogonal sub-carriers in the OFDM symbol.

Therefore, the OFDM symbol can be expressed as

\[
x(n) = \frac{1}{N} \sum_{m=0}^{N-1} \left( X_m e^{j2\pi mn/N} \right) \quad 0 \leq n \leq N - 1
\]

(1)

here, \(X_m\) are the baseband data on each sub-carrier. The digital-to-analog (D/A) converter then creates an analog time-domain signal which is transmitted through the channel.

At the receiver, the signal is converted back to a discrete N point sequence \(y(n)\), corresponding to each sub-carrier. This discrete signal is demodulated using an N-point fast Fourier transform (FFT) operation at the receiver.

The demodulated symbol stream is given by:

\[
y(m) = \sum_{n=0}^{N-1} \left( y(n) e^{-j2\pi mn/N} + W(m) \right)
\]

(2)

Here, \(0 \leq m \leq N - 1\)

where \(W(m)\) corresponds to the FFT of the samples of \(W(n)\), which is the Additive White Gaussian Noise (AWGN) introduced in the channel.

III. ICI MECHANISM

OFDM uses multiple frequency channels (sub-carriers) which are necessarily orthogonal to each other. The major problem of OFDM is frequency offset sensitivity between the transmitted and received signals or the difference between the transmitter and receiver local oscillator frequencies. Frequency offset sensitivity may be due to Doppler shift of the channel. This carrier frequency offset causes loss of orthogonality among sub-carriers and then signals carried by sub-carriers becomes dependent on each other that is some amount of overlapping occurs between adjacent channels, which leads to inter-carrier interference (ICI).

It is one of the major limitations which occur in OFDM system. OFDM is sensitive to carrier frequency offset which causes attenuation and rotation of subcarriers and majorly inter carrier interference (ICI). Because of the orthogonality of the subcarriers, we are able to extract the transmitted symbols at the receiver as they do not interfere with each other. If there is a change at receiver’s end in frequency of subcarriers due to any reason, then the orthogonality between them is lost and ICI tends
to happen. This results in heavy signal degradation. This change in frequency is known as frequency offset which are frequency mismatch between transmitter and receiver and Doppler Effect. The undesired ICI degrades the performance of system. ICI is a major drawback of OFDM based communication. Researchers have proposed various techniques to overcome the ICI in OFDM systems.

In this paper, the frequency offset is modeled as a multiplicative factor introduced in the channel, as shown in below figure

![Figure 2: Frequency Offset Model](image)

The received signal is given by

$$y(n) = x(n)e^{j2\pi n\varepsilon/N} + w(n)$$

(3)

Where $\varepsilon$ is the normalized frequency offset, and is given by $\Delta fNT_s$. $\Delta f$ is the frequency difference between the transmitted and received carrier frequencies and $T_s$ is the subcarrier symbol period. $w(n)$ is the AWGN introduced in the channel.

The channel frequency offset normalized by the subcarrier separation is $\varepsilon$ then received signal on subcarrier $k$ with frequency offset effect can be written as

$$Y(k) = X(k)S(0) + \sum_{l=0,l\neq k}^{N-1} X(l)S(l-k) + n_k$$

(4)

Here, $k = 0, 1, 2, \ldots, N-1$

Where $N$ is the total number of the subcarriers, $X(k)$ denotes the transmitted symbol ($M$ ary phase-shift keying (PSK), for example) for the $k$th subcarrier and $n_k$ is an additive noise sample. The sequence $S(l-k)$ is defined as the ICI coefficient between $l$th and $k$th subcarriers, which can be expressed as

$$S(l-k) = \frac{\sin(\pi(l+$ $\varepsilon-k))}{N\sin(\pi(l+$ $\varepsilon-k))} \exp\left(j\frac{\pi}{N}(l+$ $\varepsilon-k)\right)$$

(5)

In the above equation (4) the first term in the right-hand side represents the desired signal. Without frequency error ($\varepsilon=0$), $S(0)$ takes its maximum value $S(0)=1$. The second term is the ICI component.

IV. KALMAN FILTER DESIGN TO OFDM SYSTEM

A state space model is a mathematical model of a process, where the process state $x$ is represented by a numerical vector. A state space model actually consists of two separate models: the process model, which describes how the state propagates in time based on external influences, such as input and noise; and the measurements $y$ are taken from the process, typically simulating noisy and/or inaccurate measurements. Our objective is to design KF in the form of an equation (6), and determine a gain matrix which minimizes the mean square of the error $e^2_k$.

$$\hat{x}_{k+1} = A_f\hat{x}_k + K_fy_k$$

(6)

Some assumptions of the discrete-time KF are:

1. The state dynamics are linear and time-invariant
2. The measurement equations is linear and time-invariant
3. The noise statistics are stationary.

The matrix $A_f \in \mathbb{R}^{N\times N}$ in the KF Equation (6), The matrix $K_f \in \mathbb{R}^{P\times N}$ in the KF Equation (6) is the KG matrix which relates the estimated state $\hat{x}_k$. Note that the matrix $A_f$ & $K_f$ both are time varying matrices to be determined in order that the
estimation error $e_k = x_k - \hat{x}_k$ is guaranteed to be smaller than a certain bound for all uncertainty matrices, i.e., the estimation error dynamics satisfies equation (7).

$$E\left((x_k - \hat{x}_k)(x_k - \hat{x}_k)^T\right) \leq S_k.$$  \hspace{1cm} (7)

The matrix $S_k \in \mathbb{R}^{n \times n}$ in the Equation (7) is the covariance of estimation error. $S_k$ Being an optimize upper bound of filtering error covariance over the class of quadratic filter to be defined later.

Now, we find a solution to the Kalman filtering problem over finite horizon $[0, N]$ will be given using a Discrete Riccati Equation (DRE) approach. For the purpose, consider the augmented vector,

$$g_k = \begin{bmatrix} x_k - \hat{x}_k \\ \hat{x}_k \end{bmatrix} = \begin{bmatrix} e_k \\ \hat{e}_k \end{bmatrix} \in \mathbb{R}^{2n},$$  \hspace{1cm} (8)

$$g_{k+1} = \begin{bmatrix} x_{k+1} - \hat{x}_{k+1} \\ \hat{x}_{k+1} \end{bmatrix} = \begin{bmatrix} e_{k+1} \\ \hat{e}_{k+1} \end{bmatrix}.$$  \hspace{1cm} (9)

Using Equation (6)-(9) we get,

$$e_{k+1} = Ax_k + Bw_k - \left[ A_f \hat{x}_k + K_f C x_k + K_f v_k \right]$$  \hspace{1cm} (10)

$$e_{k+1} = Ax_k + Bw_k - A_f \hat{x}_k - K_f C x_k - K_f v_k$$  \hspace{1cm} (11)

$$e_{k+1} = \left( A - K_f C \right) x_k + Bw_k - A_f \hat{x}_k - K_f v_k$$  \hspace{1cm} (12)

$$e_i = \left( A - K_f C \right) \xi_i - \left( A - K_f C \right) \hat{x}_i + \left( A - K_f C \right) \hat{x}_i + Bw_i - A_f \hat{x}_i - K_f v_i$$  \hspace{1cm} (13)

$$\hat{x}_{k+1} = A_f \hat{x}_k + K_f C x_k + K_f C \hat{x}_k - K_f C \hat{x}_k + K_f v_k$$  \hspace{1cm} (14)

$$\hat{x}_{k+1} = A_f \hat{x}_k + K_f C x_k + K_f C \hat{x}_k - K_f C \hat{x}_k + K_f v_k$$  \hspace{1cm} (15)

$$\hat{x}_{k+1} = \left( A_f + K_f C \right) \hat{x}_k + K_f C e_k + K_f v_k$$  \hspace{1cm} (16)

Using equation (13) and (17), we get the state space equation for the estimation error $e_k$ as follows,

$$\xi_{k+1} = A_{f1} \xi_k + G \eta_k,$$  \hspace{1cm} (18)

where

$$\xi_k = \begin{bmatrix} e_k \\ \hat{e}_k \end{bmatrix}, \eta_k = \begin{bmatrix} w_k \\ v_k \end{bmatrix}, \quad A_{f1} = \begin{bmatrix} A - K_f C & A - A_f - K_f C \\ K_f C & A_f - K_f C \end{bmatrix}.$$ Let (6) be a quadratic filter associated with a guaranteed cost matrix $\Sigma_k = \Sigma_k^T \geq 0$. Then the covariance matrix of $\xi_k$ of the error system satisfying the bound,

$$\xi_k^T \Sigma_k \xi_k \leq \Sigma_k, \forall K \in [0, N]$$  \hspace{1cm} (19)

Using equation (22) we can rewrite this equation as follows,

$$\xi_{k+1}^T \Sigma_{k+1} \leq \Sigma_{k+1}$$  \hspace{1cm} (20)

Using equation (19)-(20) we get,
\[ E \left[ \{ A_k \xi_k \} \{ A_k \xi_k \}^T \right] \leq \Sigma_{k+1} \] (21)

After some algebraic manipulation we get,
\[ A_k \Sigma_k A_k^T + G W G^T \leq \Sigma_{k+1} \] (22)

Where \( \bar{W} = \text{diag} \{ W, V \} \), equation (22) is called matrix inequality equation. Next, we shall derive the necessary condition on the filter for optimality of the upper bound on the above error variance i.e. \( \Sigma_k \), \( \forall k \in [0, N] \). It will be shown that the filter (6) indeed satisfies this condition. Note that the optimal solution of Equation (22) should be of the following partitioned form:
\[
\Sigma_k = \begin{bmatrix}
E(\varepsilon_k^2) & E(\varepsilon_k \tilde{x}_k^T) \\
E(\tilde{x}_k \varepsilon_k^T) & E(\tilde{x}_k \tilde{x}_k^T)
\end{bmatrix} = \begin{bmatrix}
\Sigma_{11,k} & \Sigma_{12,k} \\
\Sigma_{21,k} & \Sigma_{22,k}
\end{bmatrix} = \begin{bmatrix}
S_k & 0 \\
0 & P_k - S_k
\end{bmatrix}
\] (23)

Our main aim is to reduce covariance of estimation error \( \Sigma_{11,k} \) so using Equation (19) and (23) we get,
\[ E(\varepsilon_k^T) \leq \Sigma_{11,k} \forall k \in [0, N] \] (24)
\[ e_k = x_k - \hat{x}_k \] (25)

Note that the Equation (23) we want to minimize only term \( A_{11} \) or we are optimizing the equation (24), so using equation (22) & (24) we get,
\[
\Sigma_{11,k+1} = \Delta K_f + \left( (A - K_f C) \left( P_k - \Sigma_{11,k} \right) \right) (A - K_f C)^T
\]
\[ - (A - K_f C) (P_k - \Sigma_{11,k}) A_f^T - A_f (P_k - \Sigma_{11,k}) (A - K_f C)^T
\]
\[ + A_f (P_k - \Sigma_{11,k}) A_f^T \]
Let
\[ Z_1 = A_f, Z_2 = (A - K_f C) \& R = (P_k - \Sigma_{11,k}) \]
So, we can rewrite equation (28) as follows,
\[ \Sigma_{11,k+1} = \Delta K_f - Z_2 R Z_1^T - Z_1 R Z_1^T + Z_2 R Z_2^T - Z_1 R Z_2^T \] (29)

Using equation (29) we get the value of \( A_f \)
\[ A_f = A - k_f C \] (30)
\[ \Delta K_f = (A - K_f C) \Sigma_{11,k} (A - K_f C)^T + (A - K_f C) \Sigma_{11,k} (A - K_f C)^T + (A - K_f C) \left( P_k - \Sigma_{11,k} \right) (A - K_f C)^T + BW B^T + K_f V K_f^T \] (31)

Now we are going to select the optimal value of \( K_f \), put the value of \( A_f \) equation (27) we get,
\[ \Sigma_{11,k+1} = \Delta K_f \] (32)
\[ \Sigma_{11,k} = (A - K_f C) \left[ \Sigma_{11,k} + P_k - \Sigma_{11,k} \right] (A - K_f C)^T + BW B^T + K_f V K_f^T \] (33)
let \( Q_k = \Sigma_{11,k} + P_k \), so we can rewrite equation (33) as follows,
\[ \Sigma_{11,k+1} = A Q_k A^T - A Q_k C^T K_f^T - K_f C Q_k A^T + K_f \left( C Q_k C^T + V \right) K_f^T + BW B^T \] (34)
let \( R = CQ_1C^T + V \), so we can rewrite equation (33) as follows,

\[
\Sigma_{11,k+1} = AQ_1A^T - AQ_1C^TK^T_f-K_fCQ_1A^T + K_fRK^T_f+ BWB^T
\]  

(35)

Let \( Z_1 = K_f \) & \( Z_2 = AQ_1C^T \)

\[
\Sigma_{11,k+1} = (Z_1-Z_2)R^{-1}(Z_1-Z_2)^T-Z_2R^{-1}Z_2^T + AQ_1A^T + BWB^T
\]  

(36)

Using equation (36) we get the value of KG as follows,

\[
K_f = (AQ_1C^T)R^{-1}
\]  

(37)

Now substituting equation (37) into (36) we get,

\[
S_{k+1} = AQ_1A^T - (AQ_1C^T)R^{-1}(AQ_1C^T)^T + BWB^T
\]  

(38)

According to all these result we can say that, the filter (6) is a quadratic estimator with an upper bound of error covariance \( S_k \).

V. TOTAL ICI CANCELLATION SCHEME

In this, a new approach to solve the ICI problem in mobile OFDM system without estimating frequency offset through training symbols (and without data rate reduction) is proposed. In this approach first we quantize the normalized frequency offset into M discrete values, leading to M spreading code matrices. Next, by decoding the received signal using these M spreading code matrices, M decisions are made on the data symbols. Using these M data symbols to recreate the received signal with ICI and measuring the Euclidean distance of the M recreated signals with the actual received signal, the best normalized frequency offset is chosen and the best corresponding data symbols are determined.

The normalized frequency offset is unknown to the receiver, we can quantize into M equally spaced values.

\[
\varepsilon_m = m \Delta \varepsilon \quad m=0, 1…M-1
\]  

(39)

Where \( \Delta \varepsilon \) is the quantization level of normalized frequency offset, and M is the number of quantization levels:

\[
\Delta \varepsilon = 1/M
\]

Hence, we have M ICI coefficient matrices

where mth matrix corresponds to

\[
\tilde{S}_m = \begin{pmatrix}
S_m(0) & S_m(-1) & S_m(1-N) \\
S_m(1) & S_m(0) & S_m(2-N) \\
S_m(N-1) & S_m(N-2) & S_m(0)
\end{pmatrix}
\]

Now, it is important to note that the ICI coefficient matrix \( S \) is an orthogonal matrix, i.e. \( SS'^* = I \)

Here \( S'^* \) is the conjugate transpose of matrix \( S \) and \( I \) is identity matrix.

Using M matrices, we can have M decisions on the transmitted data vector where the mth branch will make decision on the estimation of transmitted data vector as

\[
\hat{X}_m = \text{sgn}(\bar{R}\tilde{S}_m^*) \bar{X}
\]  

(40)

next, each branch can reproduce the received signal

\[
\hat{Y}_m = \hat{X}_m \tilde{S}_m
\]  

(41)

Hence, we only need to calculate and compare the Euclidean distances between the M reproduced received signal vectors and the truly received signal vector \( \bar{Y} \) and pick the one with the minimum distance to be the best branch and use that branch’s estimated data vector as the final decision

\[
\hat{Y}_m = \arg \min \{ ||\bar{Y}_m - \bar{Y}||^2 \}
\]  

(42)

Where \( ||\bar{Y}_m - \bar{Y}||^2 \) represents the Euclidean distance between vector \( \bar{Y}_m \) and \( \bar{Y} \). The block diagram of the Total ICI Cancellation scheme is shown.
In this paper we compare both the techniques of kalman filter and total cancellation techniques for the reduction of ICI. Using Matlab software tool the output waveform are compare with their BER. In this paper we conclude that total cancellation scheme in OFDM is efficient than all other cancellation techniques.

VII. SIMULATION RESULT

Using MATLAB software we obtain different simulation outputs using kalman filter as well as total ICI cancellation scheme for ICI reduction. Comparison for above two schemes is done here.

Following parameter specifications are used in simulation process:
1. FFT size (N): 64, 128, 256.
2. Preamble size: 256.
3. Normalized frequency offset (ε): 0.3.
4. Signal Constellation: BPSK.
5. OFDM SYMBOL for one loop: 1000.

Figure 4: BER vs SNR for general OFDM system affected by ICI (at N=256)

Figure 5: BER vs SNR for ICI affected OFDM system using Kalman algorithm (N).
CONCLUSION

Simulation results in AWGN channel confirm the superb performance of the Total ICI Cancellation scheme in the presence of frequency offset or time variations in the channel compared with General OFDM and Kalman filter scheme. In this paper, the performance of OFDM systems in the presence of frequency offset between the transmitter and the receiver has been studied in terms of the bit error rate (BER) performance. Inter-carrier interference (ICI) which results from the frequency offset degrades the performance of the OFDM system.

References


