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Mathematical Foundation in Practical Applications Related To Computer Science

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***Abstract*--** Discrete mathematics deals with several selected topics in mathematics that are essential to study of many computer science areas. Mathematical logic and Boolean algebra formed the foundation of both Mathematics and Computer Science. This review paper is discussed based on the applications of discrete mathematics in computer science such as Connectiveness, Atomic and Compound statements, Tautological implication and Equivalence. These topics are very helpful in certain practical applications related to computer science.

***Keywords*--** *Conjunction, Disjunction, Tautology, Contradiction, Negation.*

I. INTRODUCTION

Discrete mathematics is the branch of mathematics dealing with objects that can assume only distinct, separated values. The term "Discrete Mathematics" is therefore used in contrast with "Continuous Mathematics," which is the branch of mathematics dealing with objects that can vary smoothly (and which includes, for example, calculus).

The study of how discrete objects combine with one another and the probabilities of various outcomes is known as combinatorics. Other fields of mathematics that are considered to be part of discrete mathematics include graph theory and the theory of computation. Topics in number theory such as congruence's and recurrence relations are also considered part of discrete mathematics.

The study of topics in discrete mathematics usually includes the study of algorithms, their implementations, and efficiencies. Discrete mathematics is the mathematical language of computer science, and as such, its importance has increased dramatically in recent decades.

The set of objects studied in discrete mathematics can be finite or infinite. The term finite mathematics is sometimes applied to parts of the field of discrete mathematics that deals with finite sets, particularly those areas relevant to business.

Research in discrete mathematics increased in the latter half of the twentieth century partly due to the development of digital computers which operate in discrete steps and store data in discrete bits. Concepts and notations from discrete mathematics are useful in studying and describing objects and problems in branches of computer science, such as computer algorithms, programming languages, cryptography, automated theorem proving, and software development. Conversely, computer implementations are significant in applying ideas from discrete mathematics to real-world problems, such as in operations research.

II. IMPORTANCE OF DISCRETE MATHEMATICS IN COMPUTER SCIENCE

Achieving working knowledge of many principles of computer science requires mastery of certain relevant mathematical concepts and skills. For example, A grasp of Boolean algebra including DeMorgans Law is useful for understanding Boolean expressions and the basics of combinational circuits concepts surrounding the growth of functions and summations are useful for analysis of loop control.

Students learn about recursive definitions recurrence relations, analyzing recursive algorithms and writing recursive algorithms and programs together in the same course. They study matrices and matrix manipulations in conjunction with the array data structure. They learn about permutations and combinations, relations, graphs, and trees at the same time that their programming knowledge and sophistication are improving and they can do increasingly interesting programming exercises involving these concepts.

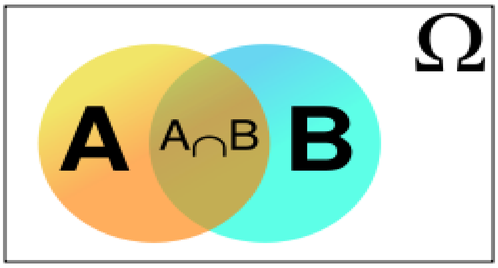
III. APPLICATIONS OF DISCRETE MATHEMATICS

A. Theoretical Computer Science

Theoretical computer science includes areas of discrete mathematics relevant to computing. It draws heavily on graph theory and logic. Included within theoretical computer science is the study of algorithms for computing mathematical results. Computability studies what can be computed in principle, and has close ties to logic, while complexity studies the time taken by computations. Automata theory and formal language theory are closely related to computability. Petri nets and process algebras are used to model computer systems, and methods from discrete mathematics are used in analyzing VLSI electronic circuits. Computational geometry applies algorithms to geometrical problems, while computer image analysis applies them to representations of images. Theoretical computer science also includes the study of various continuous computational topics.

B. Set theory

Set theory is the branch of mathematics that studies sets, which are collections of objects, such as {blue, white, and red} or the (infinite) set of all prime numbers. Partially ordered sets and sets with other relations have applications in several areas.



In discrete mathematics, countable sets (including finite sets) are the main focus. The beginning of set theory as a branch of mathematics is usually marked by Georg Cantor's work distinguishing between different kinds of infinite set, motivated by the study of trigonometric series, and further development of the theory of infinite sets is outside the scope of discrete mathematics. Indeed, contemporary work in descriptive set theory makes extensive use of traditional continuous mathematics.

C. Mathematical logic

Logic deals with all type of reasonings. These reasonings may be legal arguments or mathematical proofs or conclusions in a scientific theory.Aristotle (384 – 322BC) wrote the first treatise on logic. Gottfried Leibnitz framed the idea of using symbols in logic and this idea was realized in nineteenth century by George Bool and Augustus De Morgon. Logic is used in many branches of sciences and social sciences. It is the theoretical basis for many areas of computer science such as digital logic circuit design, automata theory and artificial intelligence. The thoughts are expressed through words. Since words have many association in every day life, there are chances of ambiguities to appear. In order to avoid this, symbols are used which have been clearly defined. Symbols are abstract and neutral. Also they are easy to write and manipulate. This is because the mathematical logic is also called symbolic logic.

Logic is the study of the principles of valid reasoning and inference, as well as of consistency, soundness, and completeness. For example, in most systems of logic (but not in intuitionistic logic) Peirce's law $(((P\to Q)\to P)\to P)$ is a theorem. For classical logic, it can be easily verified with a truth table. The study of mathematical proof is particularly important in logic, and has applications to automated theorem proving and formal verification of software.

Logical formulas are discrete structures, as are proofs, which form finite trees or, more generally, directed acyclic graph structures (with each inference step combining one or more premise branches to give a single conclusion). The truth values of logical formulas usually form a finite set, generally restricted to two values: true and false, but logic can also be continuous-valued, e.g., fuzzy logic. Concepts such as infinite proof trees or infinite derivation trees have also been studied e.g. infinitely logic.

IV. MATERIALS AND METHODS

A. Logical statement or Proposition

A statement or proposition is a sentence which is either true or false but a sentence which is both true and false simultaneously is not a statement, rather it is a paradox.

Eg: (a) Consider the following sentences:
Chennai is the capital of Tamilnadu.
The earth is the planet.
Rose is a flower.
Each of these sentences is true and so each of them is a statement.

B. Consider the following sentences:

Every triangle is an isosceles triangle.
Three plus four is eight.
The sun is a planet.
Each of these sentences is false and so each of them is a statement.

C. Consider the following sentences:

Switch on the light.
Where are you going?
How beautiful Taj Mahal is! Cannot be assigned true or false and so none of them is a statement. In fact, is a command, is a question, is exclamatory.

D. Truth value of a statement

The truth or falsity of a statement is called its truth value. If a statement is true, then its truth value is TRUE or T and if it is false , its truth value is FALSE or F. All the statements in Example 1(a) have the truth value T, while all the statements in example 1(b) have the truth value F.

E. Simple Statement

A statement is said to be simple if it cannot be broken into two or more statements. Eg: Rose is a flower.

F. Compund Statement

If the statement is the combination of two or more simple statements, then it is said to be compound statement. Eg: It is raining and it is cold.

G. Conjunction

If two simple statements p and q are connected by the word ‘and’, then the resulting compound statement ‘p and q’ is called the conjunction of p and q and is written in the symbolic form as ‘p ^ q’.

Eg: Form the conjunction of the following simple statements.

P: Ram is intelligent.

Q: Ravi is handsome.

p ^ q: Ram is intelligent and Ravi is handsome.

p	q	p^q
T	T	T
T	F	F
F	T	F
F	F	F

H. Disjunction

If two simple statements p and q are connected by the word ‘or’, then the resulting compound statement ‘p or q’ is called the disjunction of p and q and is written in the symbolic form as ‘p V q’.

Eg: Form the disjunction of the following simple statements.

P: John is playing cricket.

Q: There are thirty students in the class room.

p V q: John is playing cricket or there are thirty students in the class room.

p	q	pVq
T	T	T
T	F	T
F	T	T
F	F	F

I. Negation

The negation of a statement is generally formed by introducing the word ‘not’ at some proper place in the statement or by prefixing the statement with ‘It is not the case that’ or ‘It is false that’.

If p denotes a statement, then the negation of p is written as ~p or 7p.

Eg: p: All men are wise.

~p: Not all men are wise.

p	~p
T	F
F	T

(i)

J. Negation of a negation

Negation of a negation of a statement is the statement itself. Equivalently it is written as ~(~p)≡ p

In the truth table, the columns corresponding to p and ~(~p) are identical. Hence p and ~(~p) are logically equivalent.

p	~p	~(~p)
T	F	T
F	F	F

K. Logical Equivalence

Two compound statements A and B are said to be logically equivalent or simple equivalent, if they have identical last columns in their truth tables. In this case it is written as A ≡ B.

L. Conditional

In mathematics , we frequently come across statements of the form "If p then q". Such statements are called conditional statements or implications. The are denoted by p→q, read as 'p implies q'. The conditional p→q is false only if p is true and q is false. Accordingly, if p is false then p→q is true regardless of the truth value of q.

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

M. Bi-Conditional

If p and q are two statements, then the compound statement p→q and q→p is called bi-conditional statement and is denoted by p↔q, read as p if and only if q. p↔q has the truth value T whenever p and q have the same truth values; otherwise it is F.

p	q	p↔q
T	T	T
T	F	F
F	T	F
F	F	T

N. Tautology

A statement is said to be tautology if the last column of its truth table contains only T, i.e., it is true for all logical possibilities.

p V (~p) is a tautology.

p	~p	pV~(~p)
T	F	T
F	T	T

The last column contains only T. Therefore, p V (~p) is a tautology.

O. Contradiction

A statement is said to be contradiction if the last column of its truth table contains only F, i.e., it is false for all logical possibilities.

p ^ (~p) is a contradiction.

The last column contains only F. Therefore, p ^ (~p) is a contradiction.

p	~p	P^~(~p)
T	F	F
F	T	F

CONCLUSION

This paper emphasizes the essential role that mathematics plays in the development of computer science both for the particular knowledge and for the reasoning skills associated with mathematical maturity. The importance of certain mathematical concepts for computer science is stressed. A comprehensive table of mathematics topics and their computer science applications is presented.

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