International Journal of Trend in Research and Development, Volume 3(5), ISSN: 2394-9333
www.ijtrd.com

Hamiltonian and Eulerian Cycles

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Abstract-- In this document, examined what is issue of Hamiltonian Cycle and Eulerian cycle. Step by step instructions to make the Hamiltonian way and Eulerian way. Also, essential and adequate state of Hamiltonian cycle and Eulerian cycle calculation for study the Hamiltonian way and Eulerian way.

Here are numerous amusements and riddles which can be investigated by chart theoretic ideas. Indeed, the two early revelations which prompted the presence of diagrams emerged from puzzles, in particular, the Konigsberg Bridge Problem and Hamiltonian Game, and these riddles additionally brought about the uncommon sorts of diagrams, now called Eulerian charts and Hamiltonian charts. Because of the rich structure of these charts, they discover wide utilize both in examination and application.

Keywords-- *Graph Algorithms, Hamiltonian Cycle Problem, Eulerian Cycle Problem, Hamiltonian Path, Eulerian Path.*

I. INTRODUCTION

Given graph is called as Hamiltonian graph if it is passed on every vertex. This circuit is called as Hamiltonian circuit. And the Euler graph is called as Euler graph if it is passed on every edge. This circuit is called as Euler circuit[1].

II. HAMILTONIAN CYCLE

Hamiltonian graphs are named after the nineteenth-century Irish mathematician Sir William Rowan Hamilton (1805-1865). He uses a twenty vertices are labeled with the names of famous cities. The player is challenged to travel around the world" by finding a closed cycle along the edges of which passes through every city exactly once (this is the undirected version of the Hamiltonian cycle problem).

Hamiltonian cycle is defined as each vertex cover with only once. This is only said as a Hamiltonian cycle. A graph is called Hamiltonian graph if and only if it contains Hamiltonian cycle. Otherwise it is non Hamiltonian cycle. The issue is that to find given graph is Hamiltonian or not.

A. Definition and Problem

In the given figure, graph G (V, E), the problem is that to find Hamiltonian circuit, in the given problem contains every vertex exactly one. And it covered each vertex only once. So, this circuit is called Hamiltonian Circuit.

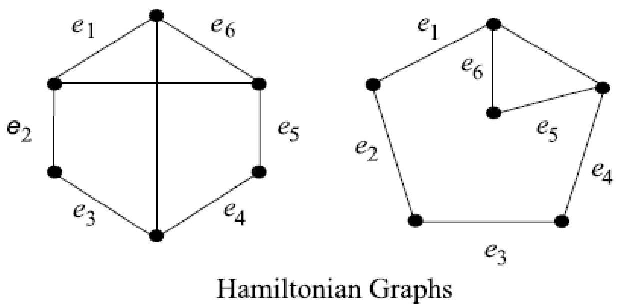


Figure 1: Hamiltonian Graph

In a figure a path that cover each vertex of the given graph once and only once that is called Hamiltonian path [2].

In a circuit that includes each vertex of the graph once and only once and it reaches to its starting point at the end and that is a called as Hamiltonian circuit [2].

By consideration, the singleton graph K is called to be Hamiltonian even though it does not haves a Hamiltonian cycle because it is include only vertex only one.

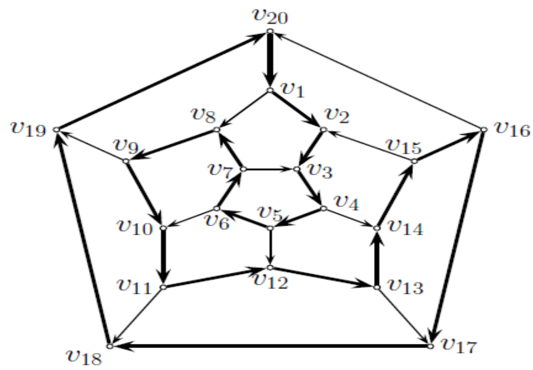


Figure 2:"A Hamilton cycle"

In graphical manner, consider that the edges between any cities, the graph shown in Figure 1 and we want to know if there is a Hamiltonian cycle in this directed graph. This is a Hamiltonian cycle is directed cycle because it include only once vertex exactly once [3].

If the last edge will be dropped the the Hamiltonian cycle is not considered as Hamiltonian cycle because it is not considered as closed loop. But it can be said that, a non-Hamiltonian graph can have a Hamiltonian path. And it is not be compulsory to each and every Hamiltonian Path has Hamiltonian cycle. For example, in Figure 3, it is shown that G1 has no Hamiltonian path; G2 has the Hamiltonian path which can be defined as v1v2v3v4, but has no Hamiltonian cycle, while G3 have Hamiltonian cycle v1v2v3v4v1 and it also have Hamiltonian Path.[3]

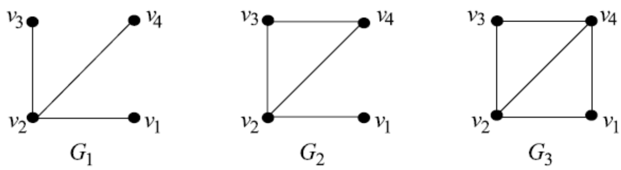


Figure 3: Hamiltonian Cycle

Some existences conditions are there that will explain for Hamiltonian path has many edges. An amazing illustration is the finished diagram Kn: it has the same number of edges as any straightforward grpah on n vertices can have, and it has numerous Hamilton cycles. The issue for a portrayal is that there are diagrams with Hamilton cycles that don't have a lot of edges. The easiest is a cycle, Cn: this has just n edges however a Hamilton cycle has. Then again, figure 2 shows diagrams with only a couple of a greater number of edges than the cycle on the same number of vertices, yet without Hamilton cycles.[4]

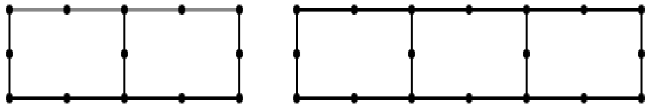


Figure 4: A graph with a Hamilton path but not a Hamilton cycle, and one with neither.

B. Necessary Condition

A must Necessary and simple condition is that any Hamiltonian graph must be strongly connected; and any undirected chart must have no cut-vertex.[5]

Assume *G=(V, E)* has more than 2 nodes.

1. No vertex of degree 1. If node *a* has degree 1, then the other endpoint of the edge incident to *a* must be visited at least twice in any circuit of *G*.
2. If a node has degree 2, then both edges incident to it must be in any Hamilton circuit.
3. No smaller circuits contained in any Hamilton circuit (the start/endpoint of any smaller circuit would have to be visited twice).
4. There must exist a subgraph of *G* with the following properties:
1. *H* contains every vertex of *G*
2. *H* is connected
3. *H* has the same number of edges as vertices
4. *H* has every node with degree 2

Then, the generated subgraph is the called as Hamilton circuit as in the given graph *G*.

In this algorithm condition four th cannot be tested in the polynomial time but the 3 rd. condition can tested fastly.

Theorem :(Dirac's Theorem): If each vertex of a diagram with three or more vertices is adjoining in any event half of the remaining vertices, then the chart has a Hamilton circuit.

So we can say that Hamiltonian is a NP-complete finished problem with the proof and condition of the therom.[4]

C. Sufficient Condition

Since, the parallel edges and loops can not be affected on Hamiltonian cycle.

Let G be a simple undirected graph of order v ≥ 3. If dG(x) + dG(y) ≥ v, for any x, y ∈ V (G), xy /∈ E(G), then G is Hamiltonian.

Proof: Assume that the theorem is false, and let G be a maximal non-Hamiltonian straightforward diagram fulfilling the condition .Since v ≥ 3.

Let the G has incomplete graph. Let x and y are a vertex that can be consider as nonadjacent vertices in G. By the consideration of G, G + xy is called as Hamiltonian. Since graph G is called as non-Hamiltonian, as called Hamilton Cycle of G+xy must contains the edge xy. Thus there is exists Hamilton path that is connecting x and y in G. Let P = (x1, x2, xv) be a Hamilton path, where x = x1 and y = xv. Set

S = {xi ∈ V (P): xxi+1 ∈ E (G), 1 ≤ i ≤ v − 2},
T = {xj ∈ V (P): yxj ∈ E(G), 2 ≤ j ≤ v − 1}.
Since G is simple and y /∈ S ∪ T, we have
|S| = dG(x), |T | = dG(y) and |S ∪ T | ≤ v − 1.[5]

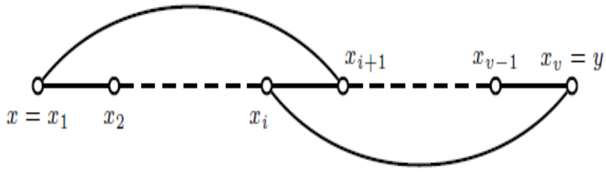


Figure 4.-Proof of theorem

D. Algorithm for Hamiltonian cycle

Optimal Hamilton Path
Input: w: matrix of weights of the graph, a: initial vertex, p: final node and N=|V|
Output: The optimal Hamilton Path
Algorithm:
Optimal Hamilton Path(w,a,p)
S: subset {1,2,…,N}
Array H [N] [N] [S]
int i,j,s,k
1. for i,j=1 to N
2. H[i,j,Æ]=w(i,j)
3. for s=1 to N-2
4. for i,j=1 to N
5. for each SÍ{[1…N]-{i,j}} with size s
6. H[i,j,S]=minkeS[d(i,k)+H(k,j,S-{k})

The complexity of this algorithm is T (2N) time. Which gives facility to work with larger graph.[6]

E. Pros and Cons

The advantage is that, the brute-force algorithm is that it is an optimal algorithm.

In the brute-force algorithm, we know about the possible output because we now where we get optimal solution.

The disadvantage is that, We have to check all the possible solution of the Hamiltonian path which is proportional to no of count of algorithm.[7]

F. Applications

Many important applications:

1. In public transport
2. Tour planning
3. In networking, design of microchips
4. genome sequencing
5. Reduction of the set of vertices.
6. Fixing of a part of the edges.[8]

III. EULERIAN CYCLE

A. Routing Problems

Routing problems are defined as a finding ways to find the delivery of goods or services to an related destinations.

The goods or services in could be packages, mail, newspapers, pizzas, garbage collection, bus service, and so on.

The delivery destinations could be homes, warehouses, distribution, centers, terminals, and the like. [1]

B. Definition and Problem

a. Euler Path

It is defined as the graph that passes each edge only once.

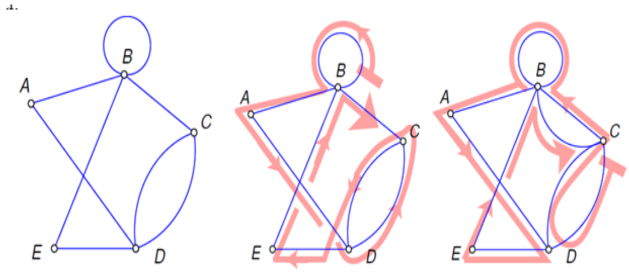
b. Euler circuit

It is defined as a it is circuit that is travels through path by every edge only once.[12]

In Euler Circuit it passes through each edge Only once and return back to its original vertex or starting point.[11]. From given definition, Euler Circuit is a subset of Euler Path.

A directed graph that travels from every edge and vertex of graph G is called an Euler graph. A closed cycle of Euler graph is called an Euler directed circuit. A circuit is called as Eulerian circuit if and only if it is contain the Eulerian path otherwise it called noneulerian.

Note: A connected graph cannot have both as a Euler circuit or Euler path, it has either Eulerian circuit or Euler path .[11]



Euler path: BBADCDEBCEuler circuit: CDCBBADEBC

Figure 5: Euler path

The Criterion for Euler Paths: Suppose that a diagram has an Euler way. For each vertex other than the beginning and consummation it, Accordingly, all vertices other than the two endpoints of P must be even vertices.
Assume the Euler way P begins at vertex x and finishes at y. At that point P leaves x once again than it enters, and leaves y one less time than it enters. Thusly, the two endpoints of P must be odd vertices.
In the event that a diagram G has an Euler way, then it must have precisely two odd vertices. On the off chance that the quantity of odd vertices in G is something besides 2, then G can't have an Euler way. [13]

c. The Criterion for Euler Circuit

Determine that a graph G has an Euler circuit C. For every vertex v the edge will be considered as once only.Vetex must be 2 degree. And the node must be even degree. Majority of its vertices having even vertices. [13]

d. Euler Graphs

Euler graph can be defined by the closed walk in the graph. It contains all the edges of graph G at once. The graph always be the connected graph. It contains also isolated vertex. But they are not contribute anything.

In the given graph below shows that clearly, v1 e1 v2 e2 v3 e3 v4 e4 v5 e5 v3 v6e7 v1 in (a) is an Euler line, because it contains every edges of the graph. While second figure (b) not a Eulerian graph because it is not have all the edges of the graph.

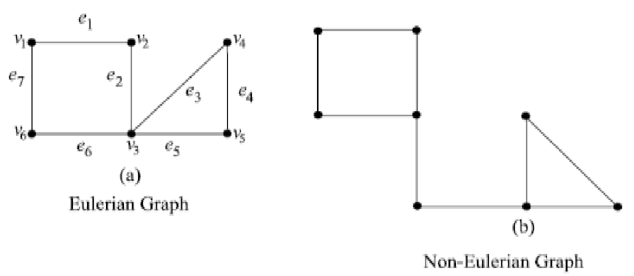


Figure 6: Eulerian and Non-Eulerian Graph

e. Theorem:

Given connected graph G is called Euler graph it must be the every vertices have even degree.[10]

f. Finding an Euler path

There are several problem that are solved by simple conditions.

- Always leave one edge available to get back to the starting vertex (for circuits) or to the other odd vertex (for paths) as the last step.
- Don't use an edge to go to a vertex unless there is another edge available to leave that vertex (except for the last step).

This method always gives output for simple graphs but it is surely need of the algorithm to define Euler circuits. [1]

C. Necessary Condition

Let G(V, E) be an Euler graph. Thus G contains an Euler line Z, which is a closed walk. Let this walk start and end at the vertex u 2 V. Since each visit of Z to an intermediate vertex v of Z contributes two to the degree of v and since Z traverses each edge exactly once, d(v) is even for every such vertex. Each intermediate visit to u contributes two to the degree of u, and also the initial and final edges of Z contribute one each to the degree of u. So the degree d(u) of u is also even.[10]

D. Sufficient Condition

Let G be a connected graph and let degree of each vertex of G be even. Assume G is not Eulerian and let G contain least number of edges. Since d _ 2, G has a cycle. Let Z be a closed walk in G of maximum length. Clearly, G−E(Z) is an even degree graph. Let C1 be one of the components of G−E(Z). As C1 has less number of edges than G, it is Eulerian and has a vertex v in common with Z. Let Z0 be an Euler line in C1. Then Z0 [Z is closed in G, starting and ending at v. Since it is longer than Z, the choice of Z is contradicted. Hence G is Eulerian.[10]

E. Algorithm for Eulerian cycle

a. Fluery's algorithm

It can be used for find for Euler circuits and paths.

FLUERY
Input: A connected graph $G = (V, E)$ with no vertices of odd degree
Output: A sequence P of vertices and their connecting edges indicating the Euler circuit.

1. Choose any vertex u0 in G.
2. $P = u0$
3. if Pi = u0e1u1e2…eiui choose edge $ei+1$ so that
   1. $ei+1$ is adjacent to ei

2. Removal of ei+1 does not disconnect $G - \{e1,\ldots,ei\}$ (i.e. $ei+1$ is not a bridge).
4. If includes all edges in E return P
5. goto 3

The algorithm for finding an Euler path is described here. Their only difference in step 1 where we must starting vertex as a odd degree vertex.

Example: The number in the given Eulerian circuit its indicated the turn of each edge.

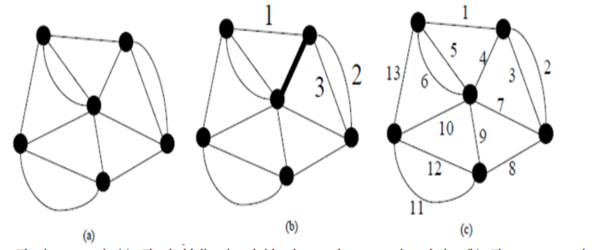


Figure: The input graph (a). The bold line is a bridge but we have no other choice (b). The output graph, showing the sequence of edges in the Euler circuit (c).

Figure 8: Euler Circuit

Figure 8 The given graph (a). The dark line indicate Euler path to find (b). The final graph, represent the flow of edges in the Euler circuit (c).[12]

b. Sub-Eulerian Graphs

A given graph G is called as a sub-Eulerian if it is a spanning a subgraph of some Eulerian graph.

The characterization of sub-Eulerian graphs is given below:

- A connected graph G is sub-Eulerian if and just if G is not spread over by acomplete bipartite cha.[13]

c. Super-Eulerian graphs

A non-Eulerian graph G is said to be super-Eulerian if it has a spanning Eulerian subgraph.

The following sufficient conditions for super-Eulerian graphs

- If a graph G is such that $n \geq 6$, $d \geq 2$ and $d(u)+d(v) \geq n-1$, for every pair of non-adjacent vertices u and v, then G is super-Eulerian.
- If G is any connected graph and if each edge of G belongs to a triangle in G, then G has a spanning Eulerian subgraph.

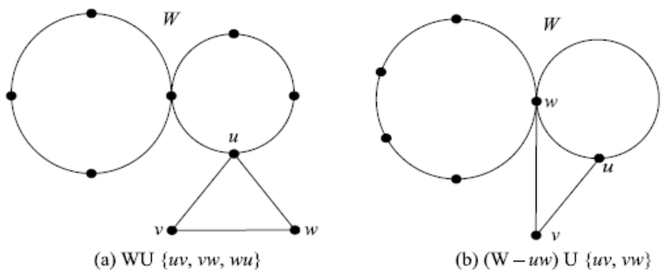


Figure 9: Super-Eulerian Graphs.

F. Applications

Eulerian graphs used to solve problem related to solve in telecommunication.

It is also used in parallel programming development and coding.

IV. COMPARISON OF EULER AND HAMILTON CIRCUITS

As of right now it must be clear that the similitude of the two issues talked about is just illusionary. There are numerous distinctions both in their down to earth apparatuses and their hypothetical examination.

Hamiltonian graph consider the each vertex while Euler graph consider each edge. The main issues illustrated is that in the Euler circuit not considered that the graph is weighted or not, while in the second one we are considered the graph is weighted or not.

The investigation illustrated that in spite of their "closeness" the Eulerian issue can be explained in direct time while a other one exists or not must be replied in NP-time ,Here is the illustrated the difference between Euler circuit and Hamiltonian circuit two issues.[12]

Euler Circuit	Hamilton Circuit
Repeated visits to a given node are allowed.	Visit each node exactly once.
Traverse each edge exactly once (by definition).	Repeated traversals of a given edge are not permitted, since that would result in visiting a node more than once.
No node may be omitted (if so, one of the edges incident to the node would not be traversed).	No node may be omitted (by definition of the problem).
No edges may be omitted (by definition)	Any edge may be omitted, as long as its endpoints can be Reached via some other points.

[12]

CONCLUSION

Henceforth, we can discover the Hamiltonian and Eulerian way by its calculation. Also, as allude to this paper we can discover Hamiltonian circuit and Eulerian circuit. it is additionally utilized as a part of different telecom framework and in system topology and it can likewise be utilized as application as a part of system security.

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