Free Vibration Analysis of Clamped Stiffened Plate by Finite Element Approach

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Abstract-- A finite element method for the free vibration characteristics of stiffened plate with central cutout, idealized with an 8-noded isoparametric quadrilateral element has been examed for clamped boundary condition by using ANSYS software package. The analysis is carried out for plate with uniform thickness, different cutout ratios, different stiffener area ratios and mode frequencies for free vibrations of stiffened plate are represented in form of non-dimensional frequency parameter.

Keywords— *ANSYS, Cutout, Free Vibration, Frequency Parameter; Plate; Stiffener.*

I. INTRODUCTION

Plates are initially flat structural elements for which the thickness is much smaller than other dimensions which develop shear forces, bending and twisting moments to resist transverse loads. Normally cutouts are provided in plates to provide easy access for maintenance, to provide access across the plate, easy assembling process, to alter the natural frequency of vibration and to avoid resonance. When interior holes were cut from plate structures, mechanical behaviors of the structures were changed. The vibration analysis of structures with cutouts possesses a tremendous challenge and must be properly understood in the structural design. The instability effects are improved with the provision of stiffeners. Stiffeners are used in cube girders, plate girders, ship hulls and wing structures.

Analysis of stiffened plates is carried out normally by energy methods by adding energies due to the plate and the stiffeners. An eight noded isoparametric stiffened plate-bending element for the free vibration analysis of stiffened plate has been presented by Mukherjee and Mukhopadhyay [13]. Here the stiffener can be positioned anywhere within the plate element and need not necessarily be placed on the nodal lines. A Buckling and vibration characteristic of stiffened plate subjected to in-plane partial edge loading has been studied by Srivastava et al.[17]. In the formulation, the stiffener can be positioned anywhere within the plate element and follow the plate beam idealization approach. Finite element analysis of eccentrically stiffened plates in free vibration has been studied by I.E. Harik and M. Guo [8]. Paramshivam P and Sridhar J.K. studied free vibration of square plate with stiffened square openings [16].

The objective of present research is to study the behavior of clamped stiffened plate subjected to free vibration idealized with an 8-noded isoparametric quadrilateral element using ANSYS software package. The analysis is carried out for plate with uniform plate thickness, different cutout ratios, different stiffener area ratios and mode frequencies for free vibrations of stiffened plate are converted to non-dimensional frequency parameter.

II. FINITE ELEMENT FORMULATION

The formulation is based on Mindlin's plate theory, which will allow for the incorporation of shear deformation.

An eight-noded isoparametric quadratic element with six degrees of freedom (u, v, w, θ_x, θ_y, θ_z) per node is employed in the present analysis. The element matrices of the stiffened plate element consist of the contribution of the plate and that of stiffener. It reveals that the contribution of beam element is reflected in all 8 nodes of the plate element, which contains the stiffener. The contribution of the stiffener to a particular node depends on the proximity of the stiffener to that node. For given edge loading and boundary condition, the static equation i.e., [K] {δ} = {F} is solved to get the stresses. The geometric stiffness matrix is now constructed with the known stresses. The overall elastic stiffness matrix, geometric stiffness matrix and mass matrix are generated from the assembly of those element matrices and stored in a single array where the variable bandwidth profile storage scheme is used. The solution of Eigen values is performed by the simultaneous iteration technique proposed by Corr and Jenning (1976).

The elastic stiffness matrix $[K_P]$, geometric stiffness matrix $[K_{GP}]$, mass matrix $[M_P]$ of the plate element may be expressed as follows

$$[K_P] = {}_{P}]^{T} [D_P]\, [B_P]\, |J_P|\, d\xi d\eta,$$

$$[K_{GP}] = {}_{GP}]^{T} [\sigma_P]\, [B_{GP}]\, |J_P|\, d\xi d\eta,$$

$$[M_P] = {}^{T} [m_p]\, [N]\, |J_P|\, d\xi d\eta.$$

The elastic stiffness matrix $[k_s]$, geometric stiffness matrix $[K_{GS}]$ and mass matrix $[M_S]$ of a stiffener element placed anywhere within plate element and oriented in the direction of x may be expressed in a manner similar to that of the plate element as follows

$$[K_S] = {}_{P}]^{T} [D_S]\, [B_S]\, |J_S|\, d\xi,$$

$$[K_{GS}] = {}_{GS}]^{T} [\sigma_S]\, [B_{\ GS}]\, |J_S|\, d\xi,$$

$$[M_S] = {}^{T} [m_s]\, [N]\, |J_S|\, d\xi,$$

$$[B_P] = [\ [B_P]_1\ \ [B_P]_2 \ .\ .\ .\ .\ \ [B_P]_r\ \ .\ .\ .\ [B_P]_8\ ]$$

$$[B_{GP}] = [\ [B_{GP}]_1\ \ [B_{GP}]_2 \ .\ .\ .\ .\ \ [B_{GP}]_r\ \ .\ .\ .\ [B_{GP}]_8\ ]$$

$$[B_S] = [\ [B_S]_1\ \ [B_S]_2 \ .\ .\ .\ .\ \ [B_S]_r\ \ .\ .\ .\ [B_S]_8\ ]$$

$$[B_{GS}] = [\ [B_{GS}]_1\ \ [B_{GS}]_2 \ .\ .\ .\ .\ \ [B_{GS}]_r\ \ .\ .\ .\ [B_{GS}]_8\ ]$$

The different matrices in the above equations may be written as follows

$$[B_S]_r = \begin{bmatrix} \frac{\partial N_r}{\partial x} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{\partial N_r}{\partial x} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\partial N_r}{\partial x} \\ 0 & 0 & \frac{\partial N_r}{\partial x} & 0 & -N_r & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The equation of equilibrium for the stiffened plate subjected to in-plane loads can be written as

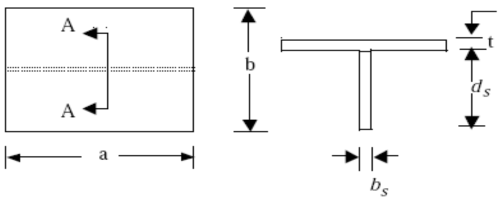
$$[M]\{\ddot{q}\} + [K_b] - P[K_G]\{q\} = 0$$

For free vibration case assuming stiffened plate vibrates harmonically with angular frequency ω, then the above equation reduces to

$$[[K_b] - P[K_G] - \omega [M]]\{q\} = 0$$

III. RESULTS AND DISCUSSION

We have considered a rectangular plate (a x b) with stiffener along with cross section is shown in Figure 1.



Section A A

Figure 1: stiffened plate cross-section

Numerical results are presented for isotropic stiffened plate with cutout for clamped boundary condition. The non-dimensional frequency parameter is given by $\lambda = \omega a^2$ where, D is plate flexural rigidity and can be calculated by the expression $D = Eh^3 / 12(1-\mu^2)$, ρ is the density of plate material and h is the plate thickness. The stiffener parameter terms δ and γ are defined as: $\delta = A_S / bh$ – ratio of cross-sectional area of the stiffener to the plate, where A_S is the area of the stiffener. $\gamma = E I_S / bD$ – ratio of bending stiffness to the plate, where I_S is the moment of inertia of the stiffener cross-section about the reference axis. g/a – ratio of cutout to the plate width.

A. Material properties

Material	Material constants		
	Young's modulus (E) in N/m^2	Density (ρ) in N/m^3	Poisson's ratio (μ)
Steel	210GPa	78500	0.3

B. Boundary condition

Boundary conditions	Position of the edge	
Clamped condition (C)	$u = w = \theta_x = \theta_y = \theta_z = 0$ at x=0 and $w = \theta_x = \theta_y = \theta_z = 0$ at x=a	$v = w = \theta_x = \theta_y = \theta_z = 0$ at y=0 and $w = \theta_x = \theta_y = \theta_z = 0$ at y=b

C. Convergence studies

In finite element analysis, it is desirable to have the convergence studies to estimate the order of mesh size to be necessary for numerical solution. For this purpose a clamped square plate subjected to free vibration are analyzed with various mesh sizes. As the convergence studies shows that a mesh size of 10X10 is sufficient enough to get a reasonable order of accuracy.

Table 1: Convergence of first three frequency parameters for clamped isotropic square plate a=600mm, b=600mm, h=6mm.

Vibration mode	Mesh size						Reference (17)
	2x2	4x4	6x6	8x8	10x10	12x12	
1	37.90	45.79	36.98	36.10	35.98	35.98	35.983
2	455.55	91.49	77.24	73.99	73.4	73.4	73.404
3	455.55	91.49	77.24	73.99	73.4	73.4	73.404

Table 2: Natural frequency parameter of clamped square plate a=600mm, b=600mm, h=6mm. with central square cutout.

Cutout size (a_1/ a)	Present method	Reference (17)	Reference (14)
0	35.98	35.981	35.985
0.5	65.403	65.715	65.27

In order to validate the results, linear fundamental frequency parameter of a clamped isotropic square plate with various sizes of cutout are computed and compared in table with reference.

The trend of results for clamped plate is shown if Fig2. For CCCC plate, there is a steep increase in natural frequency with increasing cutout size. Frequency parameter λ increases for the increase cutout sizes 0.1-0.8 for all possible modes. But for the cutout size 0.9, decrease in frequency parameter λ is observed.

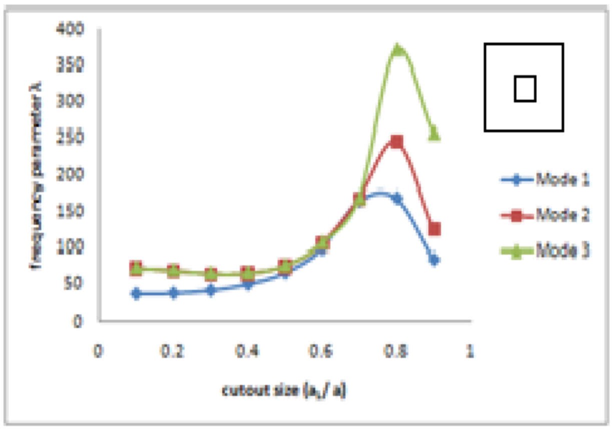


Figure 2: Variation of Frequency parameter λ v/s cutout size (a_1/ a) in various modes.

Table 3: Natural frequency ω for clamped square plate β=1, ζ=0.01 with central stiffener of δ=0.2 and γ=5

Modes	Present method	Reference (8)	Reference (13)
1	700.01	697.0	711.8
2	737.70	730.3	768.2
3	954.54	927.8	1016.5

Numerical results for frequency parameter λ for clamped stiffened square plate with central longitudinal and transverse stiffener for different plate aspect ratios β and stiffener area ratios δ are presented in Figs 3-4.

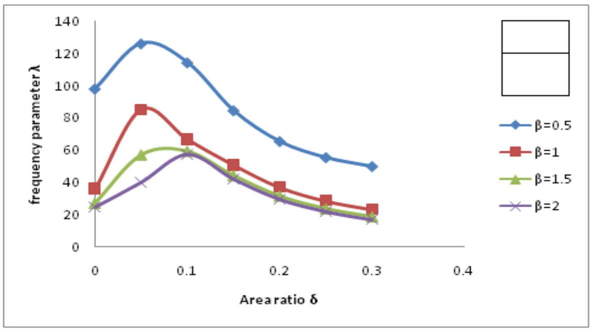


Figure 3: Variation of Frequency parameter λ with aspect ratio of plate β and area ratio of stiffener δ for centrally placed longitudinal stiffener.

It is observed that for a β= 0.5-1.5, δ= 0-0.05 and β= 2, δ= 0.1 frequency parameter λ increases. But for a β= 0.5-1.5, δ= 0-0.05 and β= 2, δ= 0.1 frequency parameter decreases gradually.

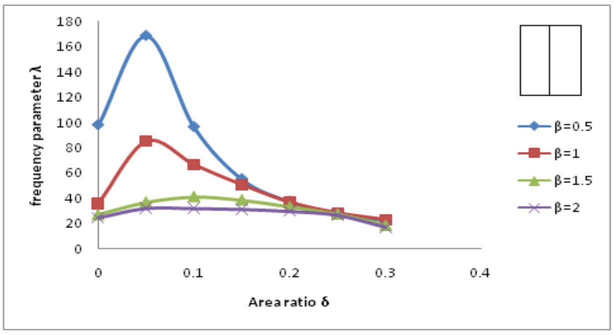


Figure 4: Variation of Frequency parameter λ with aspect ratio of plate β and area ratio of stiffener δ for centrally placed transverse stiffener.

It is observed that for a β= (0.5, 1.5, 2), δ= 0-0.05 and β= 1.5, δ= 0.1 frequency parameter λ increases. But for β= (0.5, 1.5, 2), δ= 0-0.05 and β= 1.5, δ= 0.1 frequency parameter decreases gradually.

Table 4: Natural frequency parameter for clamped stiffened square plate with cutout of (a_1/ a) = 0.2, δ= 0.15, β= 1, ζ= 0.01, γ= 5

Mode	Present method	Reference (16)	Reference (17)
1	68.45	68.62	70.57

Numerical results for frequency parameter λ for clamped stiffened perforated square plate with central longitudinal stiffener for different stiffener area ratios δ and cutout size of 0.5 are presented in Fig.5. It is observed that for δ=0-0.05 frequency parameter λ increases but for δ=0.05-0.25 frequency decreases and same variations are observed for central transverse stiffener.

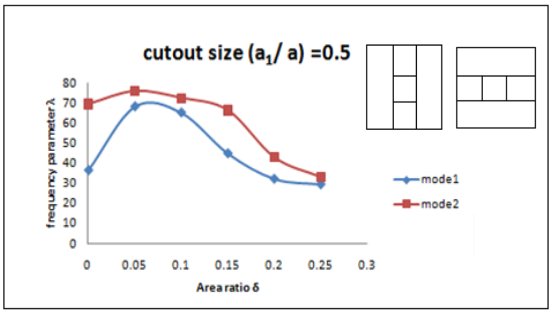


Figure 5: Variation of frequency parameter λ with area ratios δ for cutout size of 0.5.

CONCLUSION

Solutions have been obtained for the free vibration of clamped stiffened square plate with central square cutout. The following conclusions are drawn based on the present study.

1. As the aspect ratio of the plate increases the frequency parameter decreases.
2. For CCCC plate, there is a steep increase in natural frequency with increasing cutout size. Frequency parameter λ increases for the increase cutout sizes 0.1-0.8 for all possible modes. But for the cutout size 0.9, decrease in frequency parameter λ is observed.
3. In case of central longitudinally stiffened clamped plate for β= 0.5-1.5, δ= 0-0.05 and β= 2, δ= 0.1 frequency parameter λ increases. But for a β= 0.5-1.5, δ= 0-0.05 and β= 2, δ= 0.1 frequency parameter decreases gradually.
4. In case of central transversely stiffened clamped plate for β= (0.5, 1.5, 2), δ= 0-0.05 and β= 1.5, δ= 0.1 frequency parameter λ increases. But for β= (0.5, 1.5, 2), δ= 0-0.05 and β= 1.5, δ= 0.1 frequency parameter decreases gradually.
5. For clamped stiffened perforated plate It is observed that for δ=0-0.05 frequency parameter λ increases but for δ=0.05-0.25 frequency decreases and same variations are observed for central transverse stiffener.
6. Therefore for the effective reduction of frequency parameter λ, rigidity ratio γ and area ratio δ can be fixed for particular sizes of cutout.

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