

The Inspiration of Innovation Principles in TRIZ Theory to Mathematical Learning and Theoretical Innovation

¹Zhaorui Zhu and ^{2*}Hongkui Li

^{1,2}School of Mathematics and Statistics, Shandong University of Technology, Zibo, China

*Corresponding Author

Abstract: In order to respond positively to the national policy of building a basic mathematics discipline, the author looks for the commonality between the innovative methods in TRIZ theory and the learning of mathematical theoretical knowledge to inspire innovation with innovation, provide students with innovative ideas, cultivate their interest in mathematics and enhance their ability to think deeply about mathematics for better learning and research.

Keywords: Mathematics learning, TRIZ theory

Mathematics has occupied an important place in the course of human development, and the country is actively promoting the subject of basic mathematics. In fact, modern formal and exam-oriented teaching has hindered the development of students' interest in learning, resulting in students only solving problems as they see them, lacking the spirit of exploration and hindering the in-depth study of mathematical theories. TRIZ theory, however, is the summation of the knowledge and experience of countless inventors, and is a theory of innovative thinking that has been developed to increase creativity and problem-solving skills, and its application to the study of mathematics can lead to effective and in-depth learning. Therefore, this paper will explain the link between the two in detail, and use innovation to inspire innovation.

I. THE THEORETICAL ANALYSIS

1Extraction Method and Convex Function

Extraction is defined in TRIZ theory as extracting part of the need from the whole, or extracting part of the interference.

In the learning of convex functions, extraction method is a vital means to solve problems. For example, we need to extract the function value of the intermediate point to define the concavity and convexity of the function. In short, on the images of $f(x) = x^2$ and $f(x) = \sqrt{x}$, it is shown that the arc between any two points of the former is a convex function below the connection between these two points, and the latter is a concave function above the connection between any two points of the arc. But in fact, the function value of the middle point of the function arc and the middle value of the connection between the two points are compared to obtain the concave and convex. Similarly, this method is also used in the derivation of convex function properties.

Example 1 Let $f(x_0)$ be a convex function not constant and defined in (a, b) . Proof : $f(x)$ does not take the maximum value.

Lemma If $f(x)$ is a convex function on I , it must satisfy:

For any three points on $I, x_1 < x_2 < x_3$, there is always

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} \leq \frac{f(x_3) - f(x_2)}{x_3 - x_2}$$

Proof By using the counterevidence method, $f(x_0)$ can be set as the maximum value. For any $x_1, x_2 \in (a, b)$, $x_1 < x_0 < x_2$, there are

$$f(x_0) \leq \frac{x_2 - x_0}{x_2 - x_1} f(x_1) + \frac{x_0 - x_1}{x_2 - x_1} f(x_2) \leq \left(\frac{x_2 - x_0}{x_2 - x_1} + \frac{x_0 - x_1}{x_2 - x_1} \right) f(x_0)$$

Thus $f(x_0) = f(x_1) = f(x_2)$, that is, $f(x)$ is a constant function, which contradicts the meaning of the title.

This problem extracts the function values of the intermediate points of two points, which can also be expanded to extract more intermediate point function values in the learning of convex functions.

Example 2: Proving the Left and Right Derivatives of Convex Function $f(x)$ on (a, b) Based on TRIZ Theory.

Proof For any $x_0 \in (a, b)$, since $f(x)$ is a convex function on (a, b) , let $a < x_1 < x_2 < x_0 < x_3 < b$, this is

$$\frac{f(x_1) - f(x_0)}{x_1 - x_0} \leq \frac{f(x_2) - f(x_0)}{x_2 - x_0} \leq \frac{f(x_3) - f(x_0)}{x_3 - x_0}$$

This shows that the function $F(x) = \frac{f(x) - f(x_0)}{x - x_0}$ increases monotonically on (a, x_0) and has an upper bound $\frac{f(x_3) - f(x_0)}{x_3 - x_0}$ so that $\lim_{x \rightarrow x_0^-} F(x)$ exists, that is, $f'_-(x_0)$ exists, and $f'_+(x_0)$ also exists. From the arbitrariness of x_0 , we can see that $f(x)$ has left and right derivatives at (a, b) .

From these two examples, it can be concluded that in the learning of convex functions, the extraction method in TRIZ theory is used to extract the function value of the intermediate point to solve the problem, which provides a good idea for solving the problem.

II. COMBINATION AND SEQUENCE LIMIT AND FUNCTION LIMIT

The Combination method can be regarded as the combination of similar things in TRIZ theory.

In advanced mathematics learning, similar concept definitions will cause confusion in students' memory. Using the combination method of TRIZ theory to compare similar definitions and find out the inner logic is helpful for students to learn mathematical theory and improve students' understanding logical thinking ability.

For example, in the course of mathematical analysis, we can learn sequence limit and function limit by analogy.

In the limit of sequence, for convergent sequence and its limit there are Definition of $\varepsilon - N$: Let $\{a_n\}$ be an array and a be a definite number. If for any given $\varepsilon > 0$, there is always a positive integer N , so

$$\text{When } n > N, \text{ have } |a_n - a| < \varepsilon.$$

In the function limit, when x tends to x_0 , the function limit has Definition of $\varepsilon - \delta$: Let f be a function in a hollow neighborhood $U^\circ(x_0; \delta')$ of point x_0 , and A is a constant. If there is a positive number $\delta (< \delta')$ for any given positive number ε ,

$$\text{When } 0 < |x - x_0| < \delta, \text{ there is } |f(x) - A| < \varepsilon.$$

The commonness can be found from these two definitions. For the limit, a definite number needs to be found. When the unknown quantity approaches this definite number, the corresponding sequence value or function value is called as the limit.

III. ONE-DIMENSIONAL VARIABLE MULTIDIMENSIONAL METHOD AND DETERMINANT

One-dimensional to multi-dimensional method can be understood in TRIZ theory as changing the layout or action of objects from one-dimensional to multi-dimensional.

In advanced algebra courses, textbook learning focuses on the calculation of the determinant. While the calculation of the determinant alone is too abstract and vague, using the one-dimensional to multi-dimensional method to abstract the meaning of the determinant into a concrete determinant - the expansion coefficient of space, is conducive to understanding. For example, the geometric meaning of determinant of two-dimensional matrix is the area formed by two vectors; the geometric meaning of three-dimensional matrix determinant is the volume formed by three vectors. If the scaling of each dimension by the transformation is not zero, the determinant is not zero. When one-dimensional calculation becomes the area and volume in multidimensional space, the learning of determinant also expands from rigid and single calculation to tangible things in space. The combination of algebra and geometry can further promote the development of mathematical thinking.

IV. SEGMENTATION AND DIRICHLET FUNCTION

Segmentation method can be regarded as dividing objects into independent individuals in TRIZ theory

In function learning, Dirichlet function often appears as a counter-example function, and it is precisely this special identity that makes students pay special attention to when they master its nature. The value of Dirichlet function is 1 in rational number and 0 in irrational number. We use the segmentation method of TRIZ theory to approximate the Dirichlet function as the combination of $y = 1$ and $y = 0$. Similarly, when dealing with problems, Dirichlet function can be divided into rational and irrational sequences to solve problems.

Example Prove that the limit of the Dirichlet function $D(x)$ does not exist.

Proof By using the density of rational number and irrational number, we can know that for any natural number, there are rational number $x'_n \in U^\circ_-(x_0, \frac{1}{n})$, irrational number $x''_n \in U^\circ_+(x_0, \frac{1}{n})$, $n = 1, 2, 3, \dots$,

Then we can divide the rational number sequence $\{x'_n\}$ and the irrational number sequence $\{x''_n\}$ with x_0 as the limit, that is, $\lim_{x \rightarrow x_0} x'_n = \lim_{x \rightarrow x_0} x''_n = x_0$,

Let $\lim_{x \rightarrow x_0} D(x'_n) = 1$, and $\lim_{x \rightarrow x_0} D(x''_n) = 0$, known by heine theorem,

$$\lim_{x \rightarrow x_0} D(x) \text{ does not exist.}$$

It can be seen from this example that the rational number and irrational number of Dirichlet function are divided into rational number sequence and irrational number sequence by segmentation method, which creates a good situation for solving the problem.

V. NESTED METHOD AND DIFFERENTIAL MEAN VALUE THEOREM

Nested method is defined as putting one object into another in TRIZ theory.

In the study of mathematical analysis course, the differential mean value theorem is an effective tool to discuss how to infer the properties of the function from the known properties of the derivative. Rolle's theorem and Cauchy's mean value theorem are based on Lagrange's mean value theorem and derived by adding conditions, which are very similar to the nested method in TRIZ theory.

Lagrange mean value theorem: if the function f satisfies

(1) f is continuous in the closed interval $[a, b]$;

(2) f is derivable in open space (a, b) , then there is at

least one μ on (a, b) , such that

$$f'(\mu) = \frac{f(b) - f(a)}{b - a}.$$

When the Rolle theorem satisfies the conditions of (1) (3), it needs to satisfy

$$(3) f(a) = f(b)$$

Then there is at least one point μ on (a, b) , such that

$$f'(\mu) = 0$$

Cauchy mean value theorem is the generalization of Lagrange mean value theorem :

let the functions f and g satisfy (1)(2),

and at the same time, they need to satisfy that

(4) $f'(x)$ and $g'(x)$ are different from zero ;

$$(5) g(a) \neq g(b)$$

Then there is $\mu \in (a, b)$, which makes

$$\frac{f'(\mu)}{g'(\mu)} = \frac{f(b) - f(a)}{g(b) - g(a)}$$

Obviously, $g(x)$ in Cauchy mean value theorem is equivalent to x in Lagrange mean value theorem.

A new mathematical theorem is obtained by expanding the conditions of Lagrange mean value theorem. This mathematical method is worth popularizing and learning. If students use this nested method in mathematics learning, they can do more with less.

VI. REVERSE ACTION METHOD AND COUNTEREVIDENCE METHOD

Inverse action method can be defined as the opposite action in TRIZ theory.

The counterevidence method is often used in the study of mathematical theory. When we deal with problems that are difficult to solve or prove from the positive, for example, when dealing with at least at most problems, we can often choose to use the counterevidence method to reduce the difficulty of the problem.

Example Let f be derivable on R . Please prove that if the

equation $f'(x) = 0$ has no real root, then the equation $f(x) = 0$ has at most one real root.

Poof Apply proof by contradiction. If $f(x) = 0$ has two real roots x_1 and x_2 (let $x_1 < x_2$) and the function f satisfies Rolle on $[x_1, x_2]$.

So there is $\mu \in (x_1, x_2)$ so that $f'(\mu) = 0$

which contradicts the assumption of $f'(x) \neq 0$, the proposition is proved.

CONCLUSION

Through the comparative analysis of TRIZ theory and mathematical knowledge, it can be clearly found that when mathematics integrates TRIZ theory for innovative learning, the effect is often bright. Therefore, the application of TRIZ theory in mathematics learning and theoretical innovation can make students settle problems with a new perspective of development, improve innovative thinking, and enhance students' interest in mathematics learning, so as to achieve the purpose of innovation.

References

- [1] School of Mathematical Sciences, East China Normal University. Mathematical Analysis. Volume 1. 5th Edition. Beijing Higher Education Press. 2019.5
- [2] <https://mp.weixin.qq.com/s/K4lzbUK9ZUlrEbtgEiFL8g>
- [3] <https://mp.weixin.qq.com/s/1PEr2nXnwywFnxvF388KYg>
- [4] Shao Limei. Comparison of Innovation Principle in TRIZ Theory and Mathematical Theory [J]. Journal of Mudanjiang Normal University (Natural Science Edition), 2010(04):67-69.
- [5] Niu Ben, Crown Prince, Song Minghao. Mathematical Theoretical Research on the Invention Principle of TRIZ [J]. Journal of Heihe University, 2019,10(05):14-15.