International Journal of Trend in Research and Development, Volume 8(5), ISSN: 2394-9333
www.ijtrd.com

Judgement of Matrix Similarity

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Abstract: Matrix similarity is very important in matrix theory. In this paper, several methods of judging matrix similarity are given.

Keywords: *Similarity; Transitivity; Matrix equivalence; Matrix rank*

I. INTRODUCTION

Matrix similarity theory has important applications not only in various branches of mathematics, such as differential equation, probability and statistics, computational mathematics, but also in other fields of science and technology. There are many similar properties and conclusions of matrix, so it is necessary to study and sum up the similar properties and judgement of matrix.

Definition 1 Let *A* and *B* be n-order matrices, if there is an invertible matrix *P*, such that $P^{-1}AP = B$, then *A* is similar to *B*, and denoted by *A*~*B*.

Definition 2 Let *A*,be an n-order matrix, the matrix $\lambda E - A$ is called the characteristic matrix of *A*; its determinant $|\lambda E - A|$ is called the characteristic polynomial of *A*; $|\lambda E - A| = 0$ is called the characteristic equation of *A*; the root λ that satisfies the characteristic equation is called an eigenvalue of *A*; the nonzero solution vectors of the system of linear equations $(\lambda E - A)x = 0$ are called the eigenvectors of the eigenvalue λ.

Property 1(Similar transitivity of matrices) If *A* ~ *B* and *B* ~ *C*, then *A*~ *C*.

Property 2(Necessary condition for matrix similarity) if *A*~ *B*, then

1. *A* and *B* have the same characteristic polynomial;
2. *A* and *B* have the same eigenvalue;
3. *A* and *B* have the same trace;
4. *A* and *B* have the same determinant;
5. *A* and *B* have the same rank.

Note (1)The conclusions given by note property 2 are all necessary conditions. If one of the conditions is not true, we can judge that *A* and are not similar, but we can not judge that *A* and are similar.

(2) For two symmetric matrices having the same order, by **property 1**, we know that if the eigenvalues are the same, they must be similar.

Property 3(necessary and sufficient condition of similar matrix) Matrix $A \sim B \Leftrightarrow R(\lambda E - A) = R(\lambda E - A)$, that is, the characteristic matrix of *A* is equivalent to the characteristic matrix of *B*.

II. JUDGEMENT OF SIMILARITY OF MATRICES

Type 1 Using the **Definition 1** to determine that the abstract two matrices are similar.

Example 1 Let A be a 2-order matrix, $P = (\alpha, A\alpha)$ be a 2-order invertible matrix, if $A^2\alpha + A\alpha - 6\alpha = 0$, please prove $A \sim \begin{pmatrix} 0 & 6 \\ 1 & -1 \end{pmatrix}$.

Prove
$$AP = (A\alpha, A^2\alpha) = (A\alpha, 6\alpha - A\alpha)$$
$$= (\alpha, A\alpha)\begin{pmatrix} 0 & 6 \\ 1 & -1 \end{pmatrix}$$
$$= P\begin{pmatrix} 0 & 6 \\ 1 & -1 \end{pmatrix},$$

that is, $P^{-1}AP = \begin{pmatrix} 0 & 6 \\ 1 & -1 \end{pmatrix}$,by the definition, it can obtain that $A \sim \begin{pmatrix} 0 & 6 \\ 1 & -1 \end{pmatrix}$.

Type 2 For choice questions, we can using **Property 3** characteristic matrices to determine that two matrices are similar.

Example 1 Let

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}, B = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

, determine whether *A* is similar to *C* and *B* is similar to *C*?

Solution The eigenvalue of matrices *A*, *B*, and *C* are 1,2,2. Since eigenvalues 1 are single, the eigenmatrices of matrices *A*, *B*, and *C* must be equivalent. For eigenalue 2,

$$2E - A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 1 \end{pmatrix}, R(2E - A) = 1;$$

$$2E - B = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, R(2E - B) = 2;$$

$$2E-C=\begin{pmatrix}1&0&0\\0&0&0\\0&0&0\end{pmatrix},R(2E-C)=1,$$

then *A* is similar to *C*, but *B* is not similar to *C*.

Type 3 In the calculation problem, we can use **Property 1** transitivity to determine that two matrices are similar.

Example 3 Prove that n-order matrice $A=\begin{pmatrix}1&1&\cdots&1\\1&1&\cdots&1\\\vdots&\vdots&\cdots&\vdots\\1&1&\cdots&1\end{pmatrix}$ is similar to *B*= $\begin{pmatrix}0&\cdots 0&1\\0&\cdots 0&2\\\vdots&\cdots\vdots&\vdots\\0&\cdots 0&n\end{pmatrix}$.

Solution $|\lambda E-A|=|\lambda E-B|=\lambda^{n-1}(\lambda-\mathrm{n})=0$, So *A* and *B* have the same eigenvalue 0(n-1 weight) , n , and both A and B are similar to diagonal matrices $\Lambda=\begin{pmatrix}0&\cdots 0&0\\0&\cdots 0&0\\\vdots&\cdots\vdots&\vdots\\0&\cdots 0&n\end{pmatrix}$, by Property 1, we can obtain that *A* and *B* are similar.

References

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