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Mathematical Idealized Abstraction and Its Characteristic Analysis

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Abstract: Mathematics idealization abstraction is an idealized conceptual form to express the nature of the research object. It expresses the characteristics of things in a simplified and purified form, makes abstract thinking surpass reality and presents an ideal state, and constructs some idealized mathematical concepts and theories. Although some idealized abstractions cannot be achieved in practice and can only be achieved in thought, mathematical idealized abstractions play an important role in the development of mathematical theory and mathematical applications.

Keywords: *Mathematics; Idealized Abstraction; Simplification; Purification; Axiomatic System*

I. THE CHARACTERISTICS OF MATHEMATICAL IDEALIZATION AND ABSTRACTION

Idealization refers to a thinking conception of an object concept that does not exist and cannot be achieved in reality, but has its prototype in the real world. That is, the differences between different individuals in a class of things with common essential attributes constitute things with special essential attributes in thinking. Although the differences in essential attributes between different individuals are limited, the essential attributes are not in the process of idealization. Expressed in the form of infinite extremes. Idealization has already appeared in Plato's "The Utopia". Plato divides people's thinking methods into sensory thinking and rational thinking, using mathematical idealization to approximate nature. In "Das Kapital", from the many phenomena of value movement, Marx grasped the basic problem of simple commodity exchange to investigate, and idealized and abstracted the specific meanings of surplus value, profit, rent, interest, etc. developed on the basis of exchange. , Give the concept of "value".

Mathematics takes ideal existence as its research object. As in classical Greek mathematics, Ideal circle, square, The infinitely precise and limitative concepts such as points do not exist in reality. They are abstracted from the geometric and numerical depictions of the concrete facts of Athens life, and reflect reality from an idealistic level. For pure mathematical concepts, mathematical propositions, and mathematical reasoning, they do not exist concretely. They are ideal existences abstractly summarized from a large number of concrete existences. They are aimed at or refer to the characteristics or quantitative dependencies of a certain system of things, and adopt formal mathematical language. A mathematical structure expressed generally or approximately. The application of mathematics is based on actual prototypes as direct objects. It is a quantitative model that meets various practical needs. Mathematical concepts. They can be delivered to computers as mathematical software, but they also have different degrees of idealization. Therefore, the idealized abstraction of mathematics refers to the use of an idealized conceptual form to express the nature of the research object from the needs of mathematical research. There may be some properties that do not actually exist in things, but are clearly separated from reality. In the process of abstraction. They have been regarded as realistic objects for a certain need, but treated as ideal objects. If the research objects of mathematics are called quantitative models, Mathematics is the science of constructing and studying quantitative models. This quantitative model is a mathematical form that reflects the relationship structure of a class or a kind of things in accordance with certain idealized requirements. It is the common feature of such things or phenomena in terms of quantity. , Rather than the quantitative characteristics of a particular thing or phenomenon. Moreover, the idealized feature of mathematical language is also a direct reflection of mathematical thinking or the modeling of this constructive program. Because of this feature, the symbolization, formulation, and formalization of mathematical language become true.

II. THE MATHEMATICAL IDEALIZATION ABSTRACTION REVEALS THE ESSENCE OF THE PROBLEM BY SIMPLIFYING THE LAWS AMONG THE RESEARCH OBJECTS

The goal of mathematical idealization and abstraction is to make complex research objects simple. Grasp the main factor of the problem, Eliminate the interference of secondary factors, Fully expose the nature of the research object and its application rules, In principle, many features that cannot belong to its realistic prototype are cited in the connotation of each constituent concept, and idealized objects are established to replace the thought experiments conducted by objective objects. Therefore, mathematical idealization abstraction expresses the characteristics of things in simplified and purified forms, and the purpose of purification and simplification. On the one hand, it abstracts the salient features of objects so that the connections and laws between things are expressed in a simple form, revealing the hidden essence behind the phenomenon. On the other hand, make thinking go beyond reality and give full play to the freedom and flexibility of thinking and creativity.

In the process of idealized abstraction, the essence of the problem is usually presented in a simplified form of hypothesis.

First extract the common ground of things. In the process of solving the problem, it is necessary to conduct an in-depth analysis of the research object, Grasp the essence of things from the internal connections and external manifestations of the problem, Identify the same or similar relationship with existing knowledge in different objects or completely unrelated objects, avoid certain attributes of things on the basis of induction, grasp the essential characteristics of things, and use the method of idealized image to compare the objective The prototype research is simplified, the common points of things

are extracted, and ideal mathematical concepts and mathematical models are established. For example, from the objective and concrete existence of four fruits, four cars, four people, etc., the concept of natural number 4 is obtained, which is the ideal existence of mathematical thinking abstracted for the first time. And by a school, A factory, Extract their common property-point, Mathematically, "dot" is defined as having no part, No size, There is no thickness, It is the limit state of unlimited reduction in size. However, any object not only has size, but also has physical and chemical properties. Therefore, such a point does not exist in nature. The concept of point is an imaginary idealized concept abstracted through thought experiments, and geometry is correct. It is a product of the idealization of physical space from the perspective of quantity relationship and spatial form. If you do not give up "point size", "length of straight line", "thickness of plane", and ideal geometry is established, geometry cannot be obtained. development of.

The second is to choose the depth of things. A thing often has several characteristics, The essence of idealized abstraction is to select one or several characteristics that are considered to be particularly important in a certain aspect from these characteristics. Ignore other characteristics, Limit the scope of inquiry, Highlight the key points, Limit other ideas, lead a certain idea deeper, and reveal the essence of things. For example, when calculating the area of a curved trapezoid, the idealized abstraction of infinitely separable geometric figures is introduced, regardless of the molecular structure of the objective object, the curved trapezoid is continuously divided into n small curved trapezoids, and the curve is replaced by a straight line. The area of a rectangle approximately represents the area of a small curved trapezoid, and the sum of the areas of n small rectangles approximately represents the area of a trapezoid with a curved side, and the limit is taken to deduce the unachievable process from the achievable process in the limit form. In a similar way of thinking about problems, Discuss the distance traveled by an object that moves linearly at a variable speed in a certain time interval, Need to divide the time into infinite time periods, Replace variable speed with constant speed, Sum, Take the limit. So for different practical problems, Grasp the essence of solving the problem to establish the concept of points. If this method of solving the problem is extended to find the volume of the curved top cylinder, the quality of the curved and curved components, the concepts of double integral, curved integral and curved integral are further abstracted. In addition, in the research process of mathematical problems, we often use the known ideal state to simulate, superimpose simple motions to form complex motions, transform complex problems into several simple problems to solve, and study complex objects as objects with special specific properties. Wait.

III. MATHEMATICAL IDEALIZATION ABSTRACTION PURIFIES THE CHARACTERISTICS OF THE RESEARCH OBJECT TO MAKE THINKING GO BEYOND REALITY AND PRESENT AN IDEAL STATE

Mathematics idealized abstraction is to ideally reproduce the object of knowledge. The purpose is to idealize things and reappear in thinking. Due to the development of modern mathematics, The research objects of mathematics are increasingly far away from people's sensory experience, In concept, There is no and there is no channel between the law and empirical facts,Pure mathematical concepts, mathematical propositions, and mathematical reasoning are not concrete existences, but ideal existences abstractly summarized from a large number of concrete existences. Therefore, it is impossible to grasp ideal things by simply summarizing common points through observable phenomena. It is necessary to break away from the intuitive use of the abstract power of thinking to create ideal objects. Mathematicians are based on the unity and harmony formed in their long-term exploration activities The concept of simplicity embellishes and reshapes the imperfect theory provided by empirical induction to form an ideal hypothesis. Among them, axiomatic abstraction is completely idealized abstraction, It uses a set of axioms with harmony, independence and completeness to define the object, so that the apparently completely different properties and relations are only shown in different forms of the same properties and relations in the axiom system. Each theorem in the axiom system yields a series of theorems about various models of the system. Therefore, the value of the axiom system exists in the ideal model. On the one hand, the ideal model serves as the source of the given axiom system; on the other hand, the model is Application areas of the axiom system.

As Engels in "Dialectics of Nature", Take Carter's research on the basic process of a steam engine as an example, The process of idealization and abstraction is analyzed. First, suppose that under ideal conditions, the heat engine cycle is composed of two isothermal processes and two adiabatic processes. Leaving aside minor factors such as changes in the temperature of the working fluid and the exchange of heat between the working fluid and the outside world, it is proposed that the ideal cycle is reversible and Closed, irreversible factors such as friction are fundamentally ignored. After such simplification and purification, the internal process of the heat engine is revealed in a pure form, and an ideal steam engine is designed, although such an ideal cycle is impossible in reality. Really realized.

For example, the concept of group is one of the most profound and influential concepts in all mathematical concepts, which directly promotes the development of algebraic number theory, geometry, function theory, differential equations and topology. If the theory of groups consists of axioms $\varphi_1, \varphi_2, \varphi_3$ composition:

$$\varphi_1 : \forall x \forall y \forall z \ (x+y)+z = x+(y+z)$$

$$\varphi_2 : \forall x \ x+0 = x$$

$$\varphi_3 : \forall x \forall y \ x+y = 0$$

The standard model that satisfies the axiom structure is transformation group. Transformation group is connected with geometry. Geometry becomes a science that studies the invariant properties and invariants of graphics under transformation group. Projective transformation group corresponds to projective geometry, and affine transformation group corresponds to affine Geometry, similarity transformation group corresponds to Euclidean geometry. The change of parallel axioms in the axiom system of Euclidean geometry, the introduction of non-Euclidean geometry and the establishment of a modern formal axiom system, symbolize the high degree of axiomatic abstraction.

Therefore, mathematics is idealized and abstract. The thinking method is based on induction, avoiding certain attributes of things, and grasping the essential characteristics of things to establish idealized models and assumptions. This closes the relationship between various branches of mathematics. Connections promoted the development of mathematical sciences.

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