Prove the Uniqueness of Limit Convergence

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Abstract: In this paper, a new way of proving the limit uniqueness of sequence convergence is given by using the idea of set theory and convergence point, which is convenient for readers to have a deeper understanding of sequence convergence.

Keywords: *Sequence Convergence; Limit; Uniqueness; Set Theory; Convergence Point*

1 INTRODUCTION AND PREPARATORY KNOWLEDGE

An important concept in mathematical analysis is the limit convergence. In a convergent sequence, infinite numbers converge to a limit point. The limit point can accurately generalize the size of almost all terms. Many concepts in mathematical analysis can not be separated from the definition of limit. Therefore, it is of great significance to study the limit point of sequence, and the uniqueness of sequence convergence is particularly important. In this paper, we will prove the uniqueness of limit convergence in two ways, using the idea of set theory and convergence point.

Definition 1: Set { x_n } as a sequence and a as a fixed number.

If there is a positive integer N for any given positive number ε , there will always be a positive integer N, when n > N there is $|a_n - a| < \varepsilon$

If the number { x_n } sequence converges to a, the fixed number a is called the limit of the number sequence and is recorded $\lim_{n\to\infty} a_n = a$ or $a_n \to a$ ($n \to \infty$).

Definition 2: X is a set, $x \in X$ .For any given real number ε > 0, the set $\{y \in X \| x - y | < \varepsilon\}$ is denoted as B (x, ε), which is called a spherical field with X as the center and ε as the radius.

Obviously, a necessary and sufficient condition for a sequence to converge to a point is obtained.

Lemma: Let $\{x_n\}$ be a sequence and a be a fixed number. If there is always a positive integer N for any given positive number ε , so that when n > N there is B (a, ε), then { x_n } converges to a.

Definition 4: Let E be a set in R, $x \in R$. If any spherical field of X contains infinite points in E, then x is called the aggregation point of E.

2 Main conclusions

Theorem (Uniqueness) If the sequence of numbers { x_n } converges, then it has only one limit.

We prove this theorem in two ways. Firstly, we give a method to prove it in the language of set theory.

It is proved that assuming that the sequence converges to x_1 , x_2 and $x_1 \neq x_2$.

Take $\varepsilon_0 = \dfrac{|x_1 - x_2|}{2}$,

$\exists N_1$, when $n > N_1, x_n \in B(x_1, \varepsilon)$

$\exists N_2$, when $n > N_2, x_n \in B(x_2, \varepsilon)$

Order N=max $\{N_1, N_2\}$,then $x_N \in B(x_1, \varepsilon_0) \cap B(x_2, \varepsilon_0)$.

However, $B(x_1, \varepsilon_0) \cap B(x_2, \varepsilon_0) = \phi$ contradiction, so the assumption is wrong, $x_1 = x_2$.

In addition, we can also use the relevant knowledge of convergence points to prove, before that, we first give the relationship between convergence points and limit of sequence.

Theorem 1: The limit point of a sequence must be the convergence point of the sequence.

Proof:If $\{x_n\}$ a sequence converges to x, then:

$\forall \varepsilon > 0$, $\exists$ N>0, so that when n > N there is $x \in$ B (x, ε).

In this way, infinite points are found in any spherical field of x, so x is the aggregation pointof $\{x_n\}$.

However, the inverse proposition of the theorem does not hold, for example:

The sequence {1,-1, 1,-1...} is obviously not convergent, but it is convergent with 1 and-1.

So for convergent sequences, can the limit be equivalent to the convergence point? The answer is yes. We give the following theorems:

Theorem 2: The convergence point of convergent sequence must be the limit of sequence.

Proof: Assuming convergence $\{x_n\}$ to x, a is a convergence point, $x \neq a$.

Take $\varepsilon_0 = \frac{|x_1 - x_2|}{2}$, $\exists$ N > 0, so that when n > N, there is

$x_n \in B(x, \varepsilon_0)$.

But $B(x,\varepsilon_0) \cap B(a,\varepsilon_0) = \phi$

So B(ε_0 ,a) may contain only the points in the first N numbers, which is contradictory to a being the agglomeration point, so a =x.

In conclusion, we discuss the relationship between the convergence point and the limit point, and then we can directly obtain another proof of the uniqueness of the limit of a sequence by using their relationship.

It is proved that the hypothesis sequence $\{x_n\}$ converges to

x_1, x_2.

Theorem 1 knows: x_2 a convergence point of $\{x_n\}$, Theorem 2 knows: $x_2 = x_1$.

References

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