Studying Mathematical Analysis from the Perspective of Topology

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Abstract: In this paper, the knowledge of topology is used as a tool to analyze some problems involved in mathematical analysis in topological space, thus deepening the understanding of mathematical analysis and topology.

Keywords: *Mathematical Analysis; Topology; Convergence Sequence; Convergence Point; Topological Space*

I. INTRODUCTION

Topological space is a wider space. If you analyze the problems in real space in mathematical analysis, it is bound to deepen the understanding of the problems in mathematical analysis from a higher dimension. In this paper, this paper mainly studies the following questions:

1. Study the convergence sequence problem in mathematical analysis from the perspective of topology;
2. Use the concept of sequence convergence as a starting point to describe the generalization of the continuity of a function at a certain point;
3. Study the definition of the convergence point in mathematical analysis from the perspective of topology.

II. MAIN CONTENTS

In mathematical analysis, we know that if a sequence converges, then the convergence point of this sequence must be unique. However, the nature of sequence convergence in topology is very different from what we have familiar with in mathematical analysis. For example, it is easy to verify that any sequence in the mediocre space converges and converges to any point in this space, and even sequences other than the constant sequence in the discrete space have no convergence point. Thus, the convergence uniqueness theorem of the convergence sequence in the well-known mathematical analysis is not established. What is the reason for this? Topology gives an explanation: there is only one limit point for any one of the convergence sequences in the Hausdorff space. The metric space is a special Hausdorff space. In summary, the convergence uniqueness theorem in the real space of mathematical analysis is established.

In mathematical analysis, there is a theorem that uses sequence convergence to characterize a function at a point in a continuous—resolving principle, namely:

Let f(x) be defined on$U^0(x_0;\delta)$. The necessary and sufficient condition for the existence of $\lim_{x\to x_0} f(x)$is:for any sequence $\{X_n\}$contained in $U^0(x_0;\delta)$ and with x_0 as the limit, the limit$\lim_{x\to x_0} f(x_n)$exists and is equal.

Then we will think, in the general topological space, can we describe the continuity of the mapping by the nature of sequence convergence like in mathematical analysis? Unfortunately, the answer is no. We know that the sufficiency of the above-mentioned principle of conclusion is established in the topology, but the necessity is not established. For example, let X now be a real set and consider its topology as a countable complement topology. Consider the same$i: X \to R$ from the topological space X to the real space R. Since if the sequence $\{x_j\}_{j\in Z_+}$ in the topological space X converges to x_0, then there is $M \in Z_+$ such that there is $x_j = x_0$ when $j > M$, so the sequence $\{x_j\}_{j\in Z_+}$ also converges to x_0 in the real space at this time. This means that the identity definition i of the above definition satisfies the conditions behind the resolution principle. However, this identity map i is not continuous at any point in X. Because obviously any open interval U containingx_0, as long as it is not R itself, then U can not contain any open set in the topological space X, and can not be used as a neighborhood of any point. So why does the above necessity not appear in the extension space? How can we make the above theorem true? An explanation is also given in topology: the above theorem is necessary in the case of satisfying the first countable axiom. Each metric space satisfies the first countable axiom. In this way, for the real space studied in mathematical analysis, the principle of resolving is established, and the corresponding explanation is given from the perspective of topology.

In mathematical analysis we know that there are three equivalent definitions of a cluster:

Let A be a subset of the real space R, then the following conditions are equivalent,

1. x is a real number, and contains a point in set A in any neighborhood of x where x is removed;
2. Each neighborhood of x contains an infinite number of points in A;
3. There are mutually different sequences in the set $A-\{x\}$ that convergeto x.

Analysis of the above three equivalent definitions, we will find: (1) said that in any neighborhood that removes x, the point in A is included, but there is no requirement for the number of points, however, (2) requires the number of points. For an infinite number, this raises questions: Why is this happening? If we look at these three definitions from a topological point of view, we will find that these three definitions are unified: (1) is the definition of the cluster point in topology, (2) and (1) in the topological space The T1 space is equivalent, and (1) and (3) are equivalent when satisfying the first countable axiom. At the same time, any metric space is the T1 space and the space satisfying the first countable axiom, so in the real space of mathematical analysis, the above three conditions are equivalent.

CONCLUSION

From the perspective of topology, the mathematical analysis, from the original real space to look at some of the problems in the real space in a wider range of topological space, not only has a holistic understanding of the rationality and source of some problems in mathematical analysis. Moreover, it breaks the thinking mode in which the real space is solidified in our minds, and the problem is more comprehensive.

References

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