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Production and Corrective Maintenance Control of a Manufacturing System Subject to Imperfect Repairs

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Abstract: This paper carries out a joint analysis of the optimal production and corrective maintenance planning problems for a manufacturing system subject to random failures and imperfect repairs. The objective of this study is to minimize a discounted overall cost consisting of maintenance, inventory holding and backlog costs. A two-level hierarchical decision making approach, based on the determination of the expected mean time between failures (first level) and the statement of a joint optimization of production and corrective maintenance policies (second level) is proposed. Hence, the production and corrective maintenance rates are determined in the second level given the failure rates between failures, obtained from the first level. In the proposed model, the failure rate of the machine depends on the number of failures, and hence, the control policies of the considered planning problems depend on the number of failures. A computational algorithm, based on numerical methods, is used for solving the optimality conditions obtained from the application of the stochastic dynamic programming approach. Finally, a numerical example is presented to illustrate the usefulness of the proposed approach.

Keywords: *Flexible manufacturing systems; Production planning; Stochastic dynamic programming; Numerical methods; Imperfect repairs*

I. INTRODUCTION

The reliability of manufacturing systems depends on the quality of their design and on the maintenance actions that are undertaken during its operations. This paper examines the control problem of a stochastic manufacturing system consisting of one machine producing one part type. The stochastic nature of the system is attributable to the fact that the machine is subject to random breakdowns and repairs. Upon a failure of a component of the machine, an imperfect repair is undertaken. In order to consider imperfect repairs in the proposed model, the machine dynamic is assumed here to be described by a finite-state semi-Markov chain. The decisions variables are the production and the repair rates, which influence the difference between the cumulative demand and the cumulative production of finished goods. In this paper, such a difference is called a surplus. Many authors have contributed to the production planning problem of manufacturing systems, as is the case in this paper, without considering the failure rates depending on the number of failures and the control of production and corrective maintenance in the same model.

Following the work by Rishel, 1975 [1] on production planning for a system affected by jump disturbances, Xiaodong et al., 2005 [2] combined production and preventive maintenance planning in the cases where the machine's failure probability increases with its age, using the hedging point policy concept introduced by Kimemia and Gershwin, 1983 [3]. In addition, Zhang and Zheng, 2000 [4] determined a production rate and a maintenance rule minimizing the total expected cost of a two-machine system. However, with the numerical scheme adopted in their work, it remains computationally difficult to secure optimal control of a large scale manufacturing system. To cope with this difficulty, Kenne et al., 2003 [5] formulated a hierarchical control problem of production and preventive maintenance rates planning in two-machines manufacturing system, and obtained a limiting problem that was numerically more tractable. Kenne, 2006 [6] extended this production and maintenance rates control model and determined the control policy for a large case including two-tandem machine manufacturing systems.

Available works on production planning of repairable manufacturing systems have the merit of taking into account the fact that the system deteriorates with age, as real life systems do. However, following repairs or preventive maintenance, the machine is either as good as new (see Boukas and Haurie, 1990 [7]; Kenne and Boukas, 2003 [8]; Kenne et al., 2007 [9]) or is as bad as an old machine (see Nakagawa and Kowada, 1983 [10]). The inconvenience of these models is that they take into account only the extreme maintenance actions (perfect or minimal), and do not consider the real efficiency of repairs, which can significantly improve the state of the system without returning it to an as good as new position. Such a repair is called an imperfect repair; the level of repair is known as the intensity of repair, as in Nodem et al., 2009 [11], and varies from zero, for a perfect repair, to one, for minimal repair. The repair intensity could reflect the impact of repairs, and be function of the repair, as described in Love, 2000 [12], or it can be stochastic, as in Kijima, 1989 [13]. It is a function of the quality of the intervention performed, and depends on the skill level of the maintenance team as well as the number and nature of the components repaired (see Shin et al., 1996, [14]). However, Mohafid and Castanier, 2006 [15] noted that little work has been developed to take into account the case where this factor is stochastic. In deterministic cases, the repair intensity, used to model the effectiveness of corrective maintenance, is assumed to be known and constant.

It is interesting to note that Kijima, 1989 [13] proposed that upon failure, the repair undertaken could serve to reset the age of the machine only as far back as its age at the start of the last failure, called the virtual age. The literature refers to this repair model as Kijima's Type I imperfect repair model, and it has largely been used in cumulative damage models. Note that the virtual age is equal to or less than the real age, and thus, minimal repair and perfect repair are, by extension, two special cases of the imperfect repair model. Nodem et al., 2009 [11] extended Kijima's type I imperfect repair model, and determined the production rate and the repair/replacement policy that minimize the total expected cost when the system deteriorates with age, and is subject to damage failures. The aim of the proposed research is to determine a production control of a manufacturing system subject to imperfect repairs, when the failure rate increases with the number of failures.

In studying hierarchical models involving stochastic, multilevel decision processes, Dempster and Fisher, 1981 [16] argued that the objective at each level is the minimization of current costs plus the expected value of lower-level decisions, while Sethi and Zhang, 1995 [17] provided a review of several different approaches of hierarchical decision making problems in an uncertain environment. The hierarchical approach proposed in this paper consists in developing a model in which at a higher level, we determine the expected mean time between failures for a system that deteriorates with the number of failures. At a lower level, we derive a joint optimization of production and corrective maintenance for the system, which minimizes inventory, backlog, and repair costs over an infinite planning horizon. The main contribution of this work as compared to the literature is its use of a semi-Markovian process to consider the fact that failure rates depend on the number of failures. In particular, a two-level hierarchical approach is proposed, and as a result, the optimal control policy depends on the system degradation (number of failures).

The paper is organized as follows: In the next section, we present the model of the problem under consideration. In Section 3, we present the optimality conditions described by Hamilton-Jacobi-Bellman (HJB) equations and the numerical approach used to solve the (HJB) equations obtained. Then in Section 4, we present a numerical example and results. The paper is concluded in Section 5.

II. PROBLEM STATEMENT

The manufacturing system considered consists of a single machine which is subject to random breakdowns and repairs. The machine considered can produce one part type and its state can be classified as operational, denoted by 1 and under repair, denoted by 2. Let $\xi(t)$ denote the state of the machine with values in $B=\{1,2\}$. The dynamic of the machine is described by a continuous-time semi-Markov process, with a transition rate from state α to state β denoted by $\lambda_{\alpha\beta}$ with $\alpha,\beta\in B$. The transition diagram, describing the dynamic of the considered machine is presented in Fig. 1.

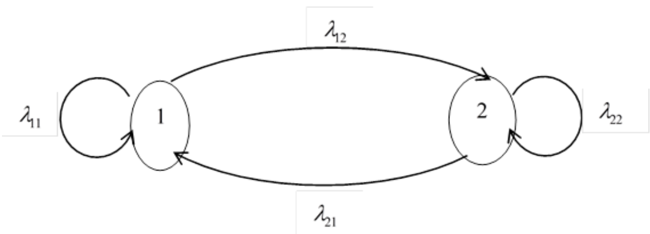

Fig. 1.States transition diagram of the considered system

The repairable systems concerned in this paper are complex and consisting of several components or subsystems. The failure of a component causes system failure and replacing or repairing the faulty component lead the system in operating state. Failure rate is deterministic and results from all interactions between the unities constituting the system.

Let T, a non-negative random variable, be the first failure time of a system with continuous density function $f(t)$. We assume that before the first failure, the failure rate λ_0 (also called the initial intensity) is a continuous function of time strictly increasing and deterministic. The failure rate of the global system at time t, over a finite planning horizon, is described by a Weibull distribution with two parameters μ and η. The density function and the failure rate function before the first failure are given by:

$$f(t)=\mu\eta^{-1}\left(\frac{t}{\eta}\right)^{\mu-1}\exp\left(-\left(\frac{t}{\eta}\right)^{\mu}\right)$$

$$\lambda_0(t)=\frac{\mu}{\eta}\left(\frac{t}{\eta}\right)^{\mu-1}$$

The Weibull law is often used in maintenance due to its flexibility to model survival times of systems and its ability to characterize their wear level through its shape parameter μ.

After system failures, imperfect repairs (between ''as good as after the previous overhaul'' and ''as good as before the overhaul or repair'') are performed to repair or replace the faulty component. The Kijima virtual age of the failed system is adjusted by a factor that reflects the degree of repair so as to bring it to a desired state somewhere between as good as new and as bad as old. The repair efficiency (improvement factor) θ is a value between 0 and 1. θ equal to 1 indicates that the component is repaired to a condition that is as good as new, while an θ equal to 0 indicates that no rejuvenation takes place after the maintenance action (minimal repair). In our study, repair efficiency used to model the effectiveness of maintenance actions, is assumed known and deterministic, it can be estimated by the maximum likelihood method based on operation data as in Shin et al., 1996 [14] and Doyen and Gaudoin, 2004 [18]. The behavior of the failure rate of the global system at time t according to the random failures, over a finite planning horizon, is determinedfor $\mu=3$,three valuesof θ (i.e. $0, 0.4, 0.7$)and for $\eta=500$.

Let $\{T_k\}\,(k\geq 1)$ be the successive failure times of a repairable system, starting from $T_0=0$. The failure rate between the k^{th} and *(k+1)*threpair is given by : $\lambda_k(t)=\lambda_{k-1}\left(t-\theta.\left(t_k-t_{k-1}\right)\right)$

when the t_k times are known and considering the conditional distributions of successive inter-failure times, this failure rate becomes:

$$\lambda_k(t)=\lambda_0\left(t-\theta.t_k\right)$$

where λ_0 is the initial intensity.

Let

$$F(t)=\int_0^t f(s)ds$$

be a distribution function of the failure time T – defined above random variable with the density function $f(t)$. Let us also define a survival function (or Reliability*)* *R(t)=1-F(t)* verifying that $\lim_{t\to\infty} t.R(t)=0$.

Consider a reparable system that is put into operation at time $t=0$ and is still functioning after the time of maintenance repair t_k. The probability that this item of age t_k survives an additional interval of length t is:

$$R(t/t_k) = P(T > t + t_k / T > t_k) = \frac{R_k(t+t_k)}{R_k(t_k)}$$

The expected remaining lifetime (Mean time between failures) after time t_k, given that the system has survived after t_k, is:

$$\mathrm{E}_k(T) = \mathrm{E}(t - t_k / t > t_k) = \int_0^\infty P(T > t + t_k / T > t_k)\,dt$$

$$E_k(T) = \int_0^\infty \frac{R_k(t+t_k)}{R_k(t_k)}dt = \frac{1}{R_k(t_k)}\int_0^\infty R_k(t+t_k)dt = \frac{1}{R_k(t_k)}\int_{t_k}^\infty R_k(t)dt$$

When lifetimes are distributed according to Weibull model

$$\mathrm{E}_k(T) = e^{(\psi_k)\mu}\left(\eta\int_0^\infty e^{-t}t^{\frac{1}{\mu}}dt - \psi_k\right)$$

where $\psi_k = \frac{(1-\theta)t_k}{\eta}$

$$\mathrm{E}_k(T) = e^{(\psi_k)\mu}\left(\eta\Gamma\left(1+\frac{1}{\mu}\right) - \psi_k\right)$$

where $\Gamma(y) = \int_0^\infty e^{-t}t^{y-1}dt$

The hierarchical approach proposed in this paper consists of a lower-level determination of the production plan of the system in operational mode, given the failure rate obtained at a higher level (Level 1 - we determine the expected mean time between failures; Level 2 - we determine the joint optimization of production and corrective maintenance policies). In order to increase the system capacity, we control the transition rate from state 2 to 1 – repair rate. Hence, the transition matrix Q depends on the corrective maintenance rate ω_r (i.e., $\lambda_{21} = \omega_r$). For the system considered, the corresponding 2×2 transition matrix Q is one of an ergodic process. Hence, $\xi(t)$ is described by the matrix $Q = \left[\lambda_{\alpha\beta}\right]$ where $\lambda_{\alpha\beta}$ verifies the following conditions:

$$\lambda_{\alpha\beta}(k,\omega_r) \ge 0 \quad (\alpha \ne \beta)$$

$$\lambda_{\alpha\alpha}(k,\omega_r) = -\sum_{\beta\ne\alpha}\lambda_{\alpha\beta}$$

The transition probabilities are given by:

$$\mathrm{P}\left[\xi(t+\delta t) = \beta \middle| \xi(t) = \alpha\right] = \begin{cases} \lambda_{\alpha\beta}(.)\delta t + o(\delta t) & if\ \alpha \ne \beta \\ 1 + \lambda_{\alpha\beta}(.)\delta t + o(\delta t) & if\ \alpha = \beta \end{cases}$$

with $\lim_{\delta t\to 0}\frac{o(\delta t)}{\delta t} = 0$ for all $\alpha, \beta \in B$

The system behavior is described by a hybrid state comprising both a discrete and a continuous component. The discrete component consists of the discrete stochastic process $\xi(t)$. Let $u(x,k,\alpha,t)$ denotes the production rate of the machine in mode α and at time t for a given surplus x and a given number of failure k. The set of the feasible control policies $\Gamma(\alpha)$, including $u(\cdot)$ and $\omega_r(\cdot)$ depends on the stochastic process $\xi(t)$ and is given by:

$$\Gamma(\alpha) = \left\{\left(u(\cdot), \omega_r(\cdot)\right) \in \Re^2, 0 \le u(\alpha,\cdot) \le u_{\max}, \omega_r^{\min} \le \omega_r(\cdot) \le \omega_r^{\max}\right\}$$

where $u(\cdot), \omega_r(\cdot)$ are known as control variables and constitute the control policy of the problem under study, $u_{\max}$ is the maximal production rate, $\omega_r^{\min}$ and $\omega_r^{\max}$ are the minimal and maximal corrective maintenance rates, respectively.

The transition rates $\lambda_{\alpha\beta}(k,\omega_r)$ of the machine after the k^{th} failure from mode α to mode β at instant t are defined by:

$$\lambda_{12}(k,\omega_r) = \frac{1}{E_k(T)} = \lim_{\delta t\to 0}\left[\frac{1}{dt}\left(p\left[\xi(t+\delta t) = 2 \mid \xi(t) = 1\right)\right]\right],$$

$$\lambda_{21}(k,\omega_r) = \omega_r(\cdot) = \lim_{\delta t\to 0}\left[\frac{1}{dt}\left(p\left[\xi(t+\delta t) = 1 \mid \xi(t) = 2\right)\right]\right]$$

Fig. 2 shows the behavior of the failure rate $\lambda_{12}(k,\cdot)$.

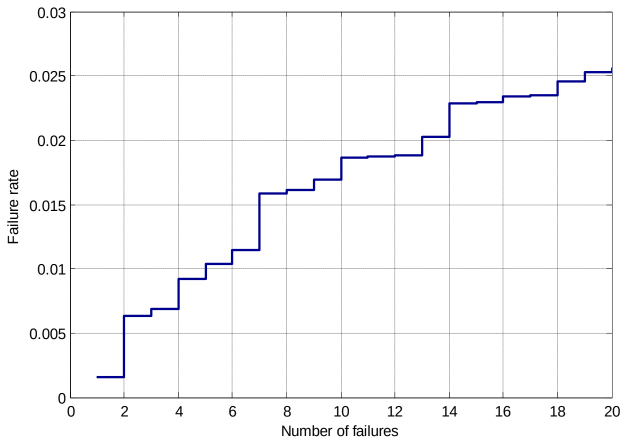

Fig. 2. Failure rate of the machine, $\theta = 0.4$

The production has to be stopped while the machine is down for corrective maintenance. The surplus may take either a positive value, called an inventory, or a negative value, called a backlog. The state equation of the surplus is given by:

$$\frac{dx(t)}{dt} = u(t) - d, \qquad x(0) = x$$

where x and d are given initial surplus and demand rate, respectively.

Let $g(\cdot)$ be the cost rate defined as follows:

$$g(\alpha, x, \cdot) = c^+x^+ + c^-x^- + c^\alpha \ , \quad \forall \alpha \in B$$

The constants c^+ and c^- are used to penalize inventory and backlog respectively; $x = \max(0, x)$, $x = \max(0, -x)$ and c^α is a constant defined as follows:

$$c^\alpha = c_r\omega_r Ind\left\{\xi(t) = 2\right\}$$

with

$$Ind\{\Theta(\cdot)\} = \begin{cases} 1 & if \quad \Theta(\cdot) \quad is \quad true, \\ 0 & otherwise. \end{cases}$$

for a given proposition $\Theta(\cdot)$. The corrective maintenance cost depends on the duration of the repair activity, described in this model by $c_r \omega_r(\cdot)$ (with $\omega_r = \omega_r^{\min}$ or $\omega_r = \omega_r^{\max}$ at the failure mode of the machine).

Our objective is to control the production rate $u(\cdot)$ and the repair rate $\omega_r(\cdot)$ such as to minimize the expected discounted cost $J(\cdot)$ given by:

$$J(\alpha, x, k, u, \omega_r) = \mathrm{E}\left\{\int_0^\infty e^{-\rho t} g(\alpha, x, \cdot) dt \,\middle|\, x(0) = x,\ \xi(0) = \alpha,\ k(t) = k\right\}$$

where ρ is the discounted rate.The value function of such a problem is defined as follows:

$$v(\alpha, x, k) = \inf_{(u(\cdot), \omega_r(\cdot)) \in \Gamma(\alpha)} J(\alpha, x, k, u, \omega_r) \qquad \forall \alpha \in B$$

In Section 3, we present the properties of the value function $v(\cdot)$ and the numerical methods used to solve the proposed optimality conditions.

III. OPTIMAL CONTROL PROBLEM AND NUMERICAL APPROACH

This section presents the HJB equations satisfied by the value function presented in (15). The properties of the value function and the method for obtaining HJB equations can be found in Kenne *et al.*, 2003 [5]. Such equations describe the optimal control policies (optimality conditions) for production and corrective maintenance planning problems. Regarding the optimality principle, we can write the HJB equations as follows:

$$\rho v(\alpha, k, x) = \min_{(u, \omega_r) \in \Gamma(\alpha)} \left[g(\alpha, x, u, \omega_r) + \sum_{\beta \in B} \lambda_{\alpha\beta} v(\beta, x) + (u - d) \frac{\partial v(\alpha, k, x)}{\partial x} \right]$$

The optimal solution, over $\Gamma(\alpha)$ of the right hand side of equation (16) is $(u^*(\cdot), \omega_r^*(\cdot))$ when the value function is available, an optimal control policy can be obtained as in the HJB equations. However, an analytical solution of the HJB equations is almost impossible to obtain. The numerical solution of the HJB equations is a challenge which was used to be considered insurmountable. Boukas and Haurie, 1990 [17] showed that implementing Kushner's method can solve such a problem in the context of production and maintenance planning.

We shall now develop the numerical methods for solving the optimality conditions given by the HJB equations. Such methods are based on the Kushner approach, such as in Hajji, et al., 2009 [19] and its references. We may recall that the main idea behind this approach consists of using an approximation scheme for the gradient of the value function $v(\alpha, k, x)$. Let h denote the length of the finite difference interval of the variable x. The value function $v(\alpha, k, x)$ is approximated by $v^h(\alpha, k, x)$, and $\dfrac{\partial v(\alpha, k, x)}{\partial x}$ is approximated using the following equation:

$$\frac{\partial v(x, \alpha)}{\partial x} \times (u - d) = \begin{cases} \dfrac{1}{h}\left(v^h(\alpha, x + h, k) - v^h(\alpha, x, k)\right) \times (u\text{-}d) & if\ \ (u\text{-}d) > 0 \\ \dfrac{1}{h}\left(v^h(\alpha, x, k) - v^h(\alpha, x - h, k)\right) \times (u\text{-}d) & otherwise \end{cases}$$

With approximations given by equation (17) and after a couple of manipulations, the HJB equations can be rewritten as follows:

$$v^h(\alpha, x, k) = \min_{(u, \omega_r) \in \Gamma^h(\alpha)} \left\{ \frac{g(\alpha, x, u, \omega_r)}{\Omega_h^\alpha (1 + \rho / \Omega_h^\alpha)} + \frac{1}{(1 + \rho / \Omega_h^\alpha)} \begin{pmatrix} p_x^{\pm}(\alpha) v^h(\alpha, x \pm h, k) \\ + \sum_{\beta \neq \alpha} p^\beta(\alpha) v^h(\alpha, x, k) \end{pmatrix} \right\}$$

where $\Gamma^h(\alpha)$ is the discrete feasible control space (or the control grid). The other terms used in the previous equation are defined as follows:

$$\Omega_h^\alpha = |\lambda_{\alpha\alpha}| + \frac{|u - d|}{h},$$

$$p_x^+(\alpha) = \begin{cases} \dfrac{u - d}{h \Omega_h^\alpha} & if \quad u - d > 0 \\ 0 & otherwise \end{cases}$$

$$p_x^-(\alpha) = \begin{cases} \dfrac{d - u}{h \Omega_h^\alpha} & if \quad u - d \le 0 \\ 0 & otherwise \end{cases}$$

$$p^\beta(\alpha) = \frac{\lambda_{\alpha\beta}}{\Omega_h^\alpha}.$$

The discrete event dynamic programming obtained can be solved using either policy improvement or successive approximation methods. The reader is referred to Kenne, et al. 2003 [5] and references therein for details on such methods.

In this paper, we use the policy improvement technique to obtain a solution for the approximating optimization problem described in this section.

IV. NUMERICAL EXAMPLE AND RESULTS

In this section, we present a numerical example and results for the manufacturing system presented in Section 2. The system capacity is described by a two-state semi-Markov process with the modes in $B = \{1, 2\}$, and the instantaneous cost described in section 2. The generator matrix $Q(\cdot)$ is explicitly defined as follows:

$$Q(k, \omega_r) = \begin{pmatrix} -\lambda_{12}(k) & \lambda_{12}(k) \\ \omega_r & -\omega_r \end{pmatrix}$$

where $\lambda_{12}(k) = 1 / E_k(T)$, with $E_k(T)$ defined in section 2.

The discrete dynamic programming (18) yields the following two equations:

$$v^h(1, x, k) = \min_{(u, \omega_r) \in \Gamma^h(1)} \left\{ \frac{c^+ x^+ + c^- x^-}{\Omega_h^1 (1 + \rho / \Omega_h^1)} + \frac{1}{(1 + \rho / \Omega_h^1)} \begin{pmatrix} p_x^{\pm}(1) v^h(1, x \pm h, k) \\ + p^2(1) v^h(2, x, k) \end{pmatrix} \right\}$$

$$v^h(2, x, k) = \min_{\omega_r \in \Gamma^h(2)} \left\{ \frac{c^+ x^+ + c^- x^- + c_r \omega_r}{\Omega_h^2 (1 + \rho / \Omega_h^2)} + \frac{1}{(1 + \rho / \Omega_h^2)} \begin{pmatrix} p_x^-(2) v^h(2, x - h, k) \\ + p^1(2) v^h(1, x, k) \end{pmatrix} \right\}$$

The computational domain D is given by:

$$D = \{(x, k) :\ -10 \le x \le 50; \quad 1 \le k \le 20\}$$

with $h = 0.25$. We may recall that the machine-mean time between failures depends on the number of failures of the machine as described by (5).

The machine will be able to meet customer demands (represented by the demand rate d), over an infinite horizon, and reach a steady state if:

$$\pi_1 * u_{\max} > d$$

where π_1 is the limiting probability at the operational mode of the machine. Note that the limiting probabilities of modes 1 and 2 (i.e., π_1 and π_2), are computed as follows:

$$\pi \cdot Q(\cdot) = 0 \quad \text{and} \quad \sum_{i=1}^{2} \pi_i = 1$$

where $\pi = (\pi_1, \pi_2)$; and $Q(\cdot)$ is the corresponding 2×2 transition rate matrix. The policy improvement technique is used to solve the obtained numerical version of the HJB equations. The results obtained are presented in Figs. 3, 4 and 5.

The production rate for $k = 5$ (i.e., for five failures of the machine), in the operational mode (i.e., mode 1) is presented in Figure 6. This figure shows that there is no need to produce parts for comfortable stock levels where the production rate is set to zero.

The production rate is therefore set to zero when the inventory is greater than 22 produced parts. The effect of large failure probabilities on large numbers of failures is minimized by assigning large values to the stock threshold, as illustrated in Figure 4. From the results obtained, the computational domain is divided into three regions, where the optimal production control policy consists of one of the following rules:

1. Set the production rate of the machine to its maximal value when the current stock level is under the threshold value, depending on the number of failures;
2. Set the production rate of the machine to the demand rate when the current stock level is equal to the threshold value;
3. Set the production rate of the machine to zero when the current stock level is higher than the threshold value.

The control policy obtained is an extension to the hedging point policy, given that the previous three rules respect the structure presented in Akella and Kumar, 1986 [20] for production planning without corrective maintenance. According to Figure 6, the production policy at mode α is given by:

$$u(x,k,1) = \begin{cases} u_{\max} & \text{if } x(\cdot) < z^*(k) \\ d & \text{if } x(\cdot) = z^*(k) \\ 0 & \text{otherwise} \end{cases}$$

where $z^*(k)$ is the optimal threshold value for each value of the machine number of failures.

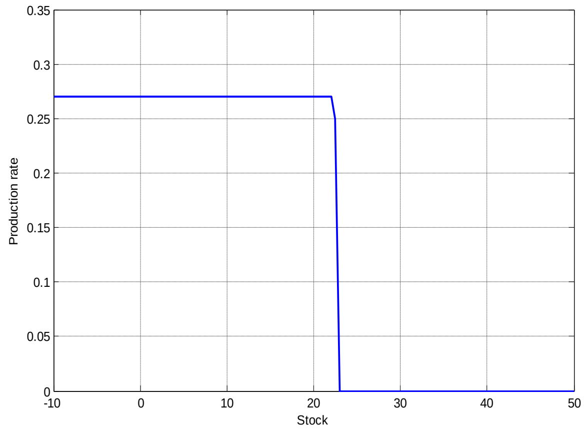

Fig. 3. Production rate at mode 1

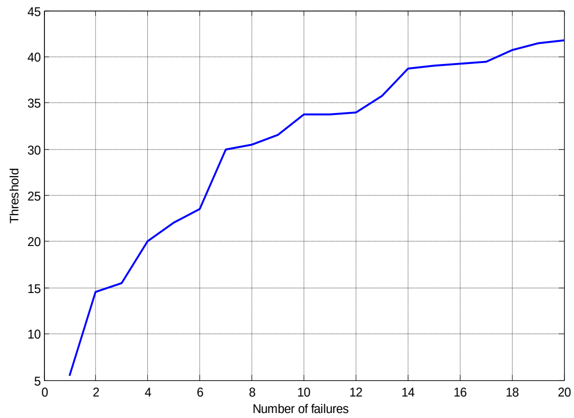

Fig. 4. Trend threshold value versus number of failures

The corrective maintenance policy, plotted in Figure 8, divides the computational domain into two regions, where the corrective maintenance rate is set to its maximal and minimal values for backlog situation (or for uncomfortable stock levels) and for large stock levels, respectively. The optimal corrective maintenance policy, like the production policy, has a bang-bang structure, and is described as follows:

$$\omega_r(x,k,2) = \begin{cases} \omega_r^{\max} & \text{if } x(\cdot) < w^*(k) \\ \omega_r^{\min} & \text{otherwise} \end{cases}$$

where $w^*(k)$ is the optimal stock level at which the corrective maintenance rate must be switched from $\omega_r^{\max}$ to $\omega_r^{\min}$.

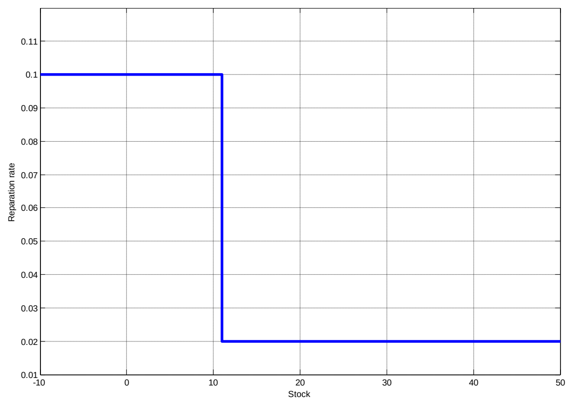

Fig. 5. Repair rate of the machine at mode 2

Using the obtained control policies, the company will minimize the total incurred cost, and thus improve its total profit. By combining k -dependent failure rate and corrective maintenance actions in production, we determined that the optimal threshold increases as the number of failures increase. We could then avoid backlogs when the machine is at mode 2. These results illustrate the contribution of the proposed model compared to one in which one value of the optimal threshold is used for production planning without considering the fact that the failure rate depends on the number of failures.

CONCLUSIONS

A hierarchical decision making approach in production and repair planning with imperfect repairs under uncertainties has been proposed. We developed the stochastic optimization model of the considered problem with two decision variables (production rate and corrective maintenance rate) and one state variable (stock level). By controlling both production and corrective maintenance rate, we obtained a near-optimal control policy for the system through numerical techniques. This work represents an important contribution to the literature on the production control of flexible manufacturing systems at several levels. Until now, production control researchers have sometimes considered that from one breakdown to the next, the machine mean time between failures is not change. In reality, however, the life cycle of the production system machines must be decreased in the long run.Our work makes it possible to consider this reality while working on machines which, following repair, are not new, with failure rates dependent on the number of breakdowns. We have shown that the number of parts to hold in inventory increases as the number of breakdowns increases, and illustrated and validated the proposed approach using a numerical example and a sensitivity analysis. The approach provides good results and extends the concept of hedging point policy to a machine-number of failures dependent production policy combined with corrective maintenance strategy. Such a policy is shown to have a bang-bang structure and is well defined if some parameters are also well defined. Based on the parameterized control policy obtained, it could be interesting to extend the proposed model to the case of manufacturing systems involving multiple products and multiple machines.

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