Performance Analysis of CRB based SB Channel Estimation Algorithm for Multiple Antenna System

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Abstract—In 4G/5G high speed systems the receiver should be able to recover the data i.e effective channel estimation is essential. Basically it is done to know or to predict the behaviour of wireless channel. We add extra bits with the data bits to know the behaviour of the channel these bits are called pilots. Now, it is achieved by using dedicated pilot symbols which consume a non negligible part of the throughput and power resources especially for large dimensional systems. The main objective of this paper is to quantify the rate of reduction of this overhead due to the use of a semi-blind channel estimation. Different data models and different pilot design schemes have been considered in this study. By using the Cramér Rao Bound (CRB) tool, the estimation error variance bounds of the pilot-based and semi-blind based channel estimators for a Multiple-Input Multiple-Output Orthogonal Frequency Division Multiplexing (MIMO-OFDM) system are compared. In particular, for large MIMO-OFDM systems, a direct computation of the CRB is prohibitive and hence a dedicated numerical technique for its fast computation has been developed. Many key observations have been made from this comparative study. The most important one is that, thanks to the semi-blind approach, one can skip about 95% of the pilot samples without affecting the channel estimation quality.

Index Terms—*Channel estimation, Semi-blind, CRB, MIMO-OFDM, large MIMO.*

I. INTRODUCTION

THE combining of the Multiple-Input Multiple-Output (MIMO) technology with the Orthogonal Frequency Di- vision Multiplexing (OFDM) (i.e. MIMO-OFDM) is widely deployed in wireless communications systems as in 802.11n wireless network [1], LTE and LTE-A [2]. Indeed, the use of MIMO-OFDM enhances the channel capacity and improves the communications reliability. In particular, it has been demonstrated in [3], [4], that thanks to the deployment of a large number of antennas in the base stations, the system can achieve high data throughput and provide very high spectral efficiency. Using multicarrier modulation techniques (OFDM in this paper) makes the system robust against frequency-selective fading channels by converting the overall channel into a number of parallel flat fading channels, which helps to achieve high data rate transmission [5]. Moreover, the OFDM eliminates the intersymbol interference and intercarrier interference thanks to the use of a cyclic prefix and an orthogonal transform. In such a system, channel estimation remains a current concern since the overall performance depends strongly on it, particularly for large MIMO systems where the channel state information becomes more challenging. Several channel estimation approaches have been developed and can be divided into two main classes. The first one concerns the blind channel estimation methods which have been extensively studied and are fully based on the statistical properties of the transmitted symbols (e.g. [6], [7], [8]). The second, used in most communications standards [1], [9], relies on inserted pilots according to a known arrangement pattern in the frame (block, comb or lattice), e.g. [10], [11]. Each channel estimation class has its own benefits and drawbacks. Generally, the second class, i.e. pilot-based channel estimator provides an easier and more accurate channel estimation than the blind estimation class. However, the second one, decreases the spectral efficiency and the throughput as compared to the first. Therefore, it would be advantageous to retain the benefits of the two techniques through the use of semi-blind estimation methods [12], [13], [14] which exploit both data and pilots to achieve the desired channel identification. This paper is dedicated to the comparative performance bounds analysis of the semi-blind channel estimation and the data-aided approaches in the context of MIMO-OFDM systems. To obtain general comparative results independent from specific algorithms or estimation methods, this analysis is conducted using the estimation performance limits given by the CRB1. Therefore, we begin by providing several CRB derivations for the different data models (Circular Gaussian (CG), Non Circular Gaussian (NCG), Binary/Quadratic Phase Shift Keying (BPSK/QPSK)) and different pilot design schemes (block, comb and lattice). For the particular case of large dimensional MIMO systems, we have exploited the block diagonal structure of the covariance matrices to develop a fast numerical technique that avoids the prohibitive cost and the out of memory problems (due to the large matrix sizes) of the CRB computation. Moreover, for the BPSK/QPSK case, a realistic approximation of the CRB is introduced to avoid heavy numerical integral calculations. After computing all the needed CRBs, It is well known that semi-blind techniques can help reduce the pilot size or improve the estimation quality [17]. However, to the best of our knowledge, this is the first study that thoroughly quantifies the achievable rate of pilot compression allowed by the use of a semi-blind approach in the context of MIMO-OFDM. A main outcome of this analysis is that it highlights the fact that, by resorting to the semi-blind estimation, one can get rid of most of the pilot samples without affecting the channel identification quality. Also an important by-product of this study is the possibility to easily design semi-orthogonal pilot sequences in the large dimensional MIMO case thanks to their significant shortening. This paper is organized as follows. Section II introduces the basic concepts and data models of the MIMO-OFDM system. Section III briefly introduces the well known pilot-based channel estimation CRB while section IV derives the analytical expressions of the semi-blind CRBs when block-type pilot arrangement is considered. Section V investigates the CRB for semi-blind channel estimation for comb-type and lattice- type pilots arrangement. The large MIMO computational issue is considered in section VI, where a new vector representation and treatment for the fast manipulation of block diagonal matrices are proposed. Section VII analyzes the throughput gain of the semi-blind channel estimation as compared to pilot-

based channel estimation. Finally, discussions and concluding remarks are drawn in section VIII.

II. MUTLI-CARRIER COMMUNICATIONS SYSTEMS: MAIN CONCEPTS

A. Main pilot arrangement patterns

Most wireless communications standards specify the insertion of training sequences (i.e. preamble) in the physical frame. These sequences are considered as OFDM pilot symbols and are known both by the transmitter and receiver. Therefore the receiver exploits these pilots to estimate the propagation channel. These pilots can be arranged in different ways in the physical frame. This paper focuses on three pilot patterns mainly adopted in communications systems. They are described in what follows.
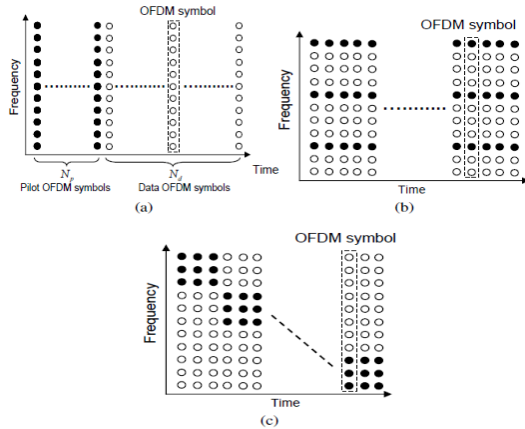

Fig. 1: Pilot arrangements: (a) Block-type with Np pilot OFDM symbols and Nd data OFDM symbols; (b) Comb type with Kp pilot sub-carriers and Kd data sub-carriers; and (c) Lattice-type with Kp pilot sub-carriers and Kd data subcarriers with time varying positions.

Fig. 1a illustrates the block-type pilot arrangement where the pilot OFDM symbols are periodically transmitted. This structure is well adapted to frequency-selective channels.

Fig. 1b concerns the comb-type pilot arrangement which is more adapted to fast fading channels. For this structure, specific and periodic sub-carriers are reversed as pilots in each OFDM symbol. Each OFDM symbol contains K_p sub-carriers dedicated to pilots and the remaining i.e. $K_d = K - K_p$ sub-carriers are dedicated to the data. Every OFDM symbol has pilot tones at the periodically-located sub-carriers.

Fig 1c represents a lattice-type pilot arrangement. In this structure the K_p sub-carrier positions are modified across the OFDM symbols in a diagonal way with a given periodicity. This arrangement is appropriate for time/frequency-domain interpolations for channel estimation. To be adapted to these two last pilot structures, equations (4) and (5) representing the MIMO-OFDM system model are modified as follows.

$$y = [\lambda_p \quad \lambda_d] \begin{bmatrix} x_p \\ x_d \end{bmatrix} + v = \begin{bmatrix} \bar{X}_p \\ \bar{X}_d \end{bmatrix} h + v \qquad (1)$$

where x_p and x_d represent the pilot and data symbol vectors, respectively. Similarly λ_p and λ_d are the corresponding system matrices.

In the sequel to take into account the time index (ignored in equation (1)), we will refer to the t-th OFDM symbol by y(t) instead of y.

III. CRB FOR BLOCK-TYPE PILOT-BASED CHANNEL ESTIMATION

The section introduces the well known analytical CRB bound [16] associated to the pilot-based channel estimation with the block-type pilot arrangement. The CRB is obtained as the inverse of the Fisher Information Matrix (FIM) denoted $J_{\theta\theta}^p$ where θ is the unknown parameter vector i.e. θ = h.

Since the noise in an independent indentically distributed (i.i.d.) random process, the FIM for θ, when N_p pilot OFDM symbols of power σ_p^2 are used, can be expressed as follows:

$$J_{\theta\theta}^p = \sum_{i=1}^{N_p} J_{\theta\theta}^{p_i} \qquad (2)$$

where $J_{\theta\theta}^{p_i}$ is the FIM associated with the i-th pilot OFDM symbol given by [17], [18]:

$$J_{\theta\theta}^{p_i} = E\left\{ \left(\frac{\partial \ln p(y(i),h)}{\partial \theta *} \right) \left(\frac{\partial \ln p(y(i),h)}{\partial \theta *} \right)^H \right\}, \quad (3)$$

where E(.) is the expectation operator, and p$(y(i),h)$ is the probability density function of the received signal given h. According to the complex derivation $\frac{\partial}{\partial \theta *} = \frac{1}{2}\left(\frac{\partial}{\partial \alpha} + j \frac{\partial}{\partial \beta} \right)$ for θ = α + jβ, the derivation of equation (8) is then expressed by:

$$J_{\theta\theta}^{p_i} = \frac{\bar{X}(i)^H \bar{X}(i)}{\sigma_v^2} \qquad (4)$$

Therefore the lower bound, denoted CRB$_{OP}$ (OP stands for 'only Pilot'), of the unbiased MSE (Mean Square Error) channel estimation when only pilots are exploited to estimate the channel is given by $\left(\bar{X}_p = \left[\bar{X}(1)^T \dots \bar{X}(N_p)^T \right]^T \right)$:

$$CRB_{OP} = \sigma_v^2 tr\left\{ \left(\bar{X}_p^H \bar{X}_p \right)^{-1} \right\} \qquad (5)$$

The best performance is reached when the pilot sequences are orthogonal as designed in [1], [19], in which case, $\bar{X}_p^H \bar{X}_p$ is simplified as follows $\bar{X}_p^H \bar{X}_p = N_p \sigma_p^2 I_{N_t N_r N}$

IV. CRB FOR SEMI-BLIND CHANNEL ESTIMATION WITH BLOCK-TYPE PILOTS ARRANGEMENT

This section addresses the derivation of the CRB analytical expression for semi-blind channel estimation when the pilot arrangement pattern is assumed to be a block-type one. In this context the CRB computation relies not only on the known

transmitted pilot OFDM symbols (i.e. training sequences) but also on the unknown transmitted OFDM symbols.

To derive the CRB expression, three cases have been considered depending on whether the transmitted data is stochastic Circular Gaussian (CG)[4], stochastic Non-Circular Gaussian (NCG) or i.i.d. BPSK/QPSK signals. Data symbols and noise are assumed to be both i.i.d. and independent. Therefore the FIM, denoted $J_{\theta\theta}$, is divided into two parts:

$$J_{\theta\theta} = J_{\theta\theta}^{p} + J_{\theta\theta}^{d}, \qquad (6)$$

where $J_{\theta\theta}^{p}$ is related to the FIM associated with known pilots (given by (7) and (9)), and $J_{\theta\theta}^{d}$ concerns the FIM dedicated to the unknown data. Depending on the data model, the vector of unknown parameters θ is composed of complex and real parameters (i.e. θ_c and θ_r) as follows:

$$\theta = \left[\theta_c^T \left(\theta_c^*\right)^T \theta_r^T\right]^T, \qquad (7)$$

where θ_c represents the complex-valued channel taps with θ_r concerns the unknown data and noise parameters. The FIM for a complex parameter θ is derived in [21], [22]. With respect to this new parameter vector, the previous pilot-based FIM matrix is expressed as:

$$J_{\theta\theta}^{p} = \begin{bmatrix} \dfrac{\bar{X}_p^H \bar{X}_p}{\sigma_v^2} & 0 & 0 \\ 0 & \dfrac{\bar{X}_p^H \bar{X}_p}{\sigma_v^2} & 0 \\ 0 & 0 & J_{\theta_r\theta_r}^{p} \end{bmatrix} \qquad (8)$$

where $J_{\theta_r\theta_r}^{p}$ will be specified later for each considered data model.

Before proceeding further, let us introduce the following notation: Denote x the signal composed of known pilots x_p and unknown transmitted data x_d: $x = \left[x_p^T x_d^T\right]^T$. The unknown transmitted data x_d is composed of N_d OFDM symbols i.e. $x_d = \left[x_{S1}^T x_{S2}^T \cdots x_{SN_d}^T\right]^T$.

A. Circular Gaussian data model $\left(\mathbf{CRB_{SB}^{CG}}\right)$

In this section, the N_d unknown data OFDM symbols are assumed to be stochastic CG and i.i.d. with zero mean and a covariance matrix C_x = diag $\left(\sigma_x^2\right)$ with $\sigma_x^2 \underline{\underline{def}} \left[\sigma_{x_1}^2 \cdots \sigma_{x_{N_t}}^2\right]^T$ where $\sigma_{x_i}^2$ denotes the transmit power of the i-th user. The data FIM is equal to the FIM of the first data OFDM symbol multiplied by the number of symbols N_d. The observed OFDM symbol is CG, i.e. y ~ NC $(0, C_y)$, where the output auto-covariance matrix is given by:

$$C_y = \sum_{i=1}^{N_t} \sigma_{x_i}^2 \lambda_i \lambda_i^H + \sigma_v^2 I_{KN_r} \qquad (9)$$

The unknown parameters θ_c and θ_r of the vector θ in equation (12) are given by:

$$\theta_c = h; \quad \theta_r = \left[\sigma_x^{2T} \sigma_v^2\right]^T \qquad (10)$$

For the pilot-based FIM, the sub-matrix $J_{\theta_r\theta_r}^{p}$ is provided by:

$$J_{\theta_r\theta_r}^{p} = \begin{bmatrix} 0_{N_t \times N_t} & 0_{N_t \times 1} \\ 0_{1 \times N_t} & \dfrac{N_r K}{2\sigma_v^4} \end{bmatrix} \qquad (11)$$

The data-based FIM of this model is given by [18]:

$$J_{\theta\theta}^{d} = \text{tr}\left\{C_y^{-1} \frac{\partial C_y}{\partial \theta^*} C_y^{-1} \left(\frac{\partial C_y}{\partial \theta^*}\right)^H\right\} \qquad (12)$$

The derivation of the FIM is related to the following equations: $\dfrac{\partial C_y}{\partial \sigma_{x_i}^2} = \dfrac{1}{2}\lambda_i\lambda_i^H$; $\dfrac{\partial C_y}{\partial \sigma_v^2} = \dfrac{1}{2}I_{KN_r}$; and $\dfrac{\partial C_y}{\partial h_i^*} = \lambda C_x \dfrac{\partial \lambda^H}{\partial h_i^*}$. To simplify the latter, for each i = 1, …, N_t; i_{N_r} = 1, …, N_r; and i_N = 1, …, N are calculated. Therefore after some simplifications, we obtain $\dfrac{\partial C_y}{\partial h_i^*} = \sigma_{x_{i_{N_t}}}^2 \lambda_{iN_L} \dfrac{\partial \lambda_{i_{N_t}}^{\Pi}}{\partial h_i^*}$. The FIM $J_{\theta\theta}^{d}$ has he following form:

$$J_{\theta\theta}^{d} = N_d \begin{bmatrix} J_{hh} & j_{hh^*} & J_{h\sigma_x^2} & J_{h\sigma_v^2} \\ J_{h^*h} & J_{h^*h^*} & J_{h^*\sigma_x^2} & J_{h^*\sigma_v^2} \\ J_{\sigma_x^2 h} & J_{\sigma_x^2 h^*} & J_{\sigma_x^2 \sigma_x^2} & J_{\sigma_x^2 \sigma_v^2} \\ j_{\sigma_v^2 h} & J_{\sigma_v^2 h^*} & J_{\sigma_v^2 \sigma_x^2} & J_{\sigma_v^2 \sigma_v^2} \end{bmatrix}, \qquad (13)$$

where

$$\left[J_{hh}\right]_{i,j} = \left[J_{h^*h^*}\right]_{i,j}^H = \quad 1 \le i, j \le N_t N_r N$$

$$\text{tr}\left\{C_y^{-1}\sigma_{x_{i_{Nt}}}^2 \lambda_{i_{Nt}} \frac{\partial \lambda_{i_{Nt}}^H}{\partial h_i^*} C_y^{-1}\sigma_{x_{i_{Nt}}}^2 \frac{\partial \lambda_{jN_t}}{\partial h_j} \lambda_{jN_t}^H\right\}, \quad (14)$$

$$\left[J_{hh^*}\right]_{i,j} = \left[J_{h^*h}\right]_{i,j}^H =$$

$$\text{tr}\left\{C_y^{-1}\sigma_{x_{i_{Nt}}}^2 \lambda_{i_{Nt}} \frac{\partial \lambda_{i_{Nt}}^H}{\partial h_i^*} C_y^{-1}\sigma_{x_{J_{Nt}}}^2 \lambda_{jN_t} \frac{\partial \lambda_{jN_t}^H}{\partial h_j^*}\right\}, \quad (15)$$

$$\left[J_{\sigma_x^2\sigma_x^2}\right]_{i,j} = \frac{1}{4}\text{tr}\left\{C_y^{-1}\lambda_i\lambda_i^H C_y^{-1}\lambda_j\lambda_j^H\right\}, 1 \le i, j \le N_t \quad (16)$$

$$J_{\sigma_v^2\sigma_v^2} = \frac{1}{4}\text{tr}\left\{C_y^{-1}C_y^{-1}\right\}, \qquad (17)$$

$$\left[J_{h\sigma_v^2}\right] = \left[J_{hh^*\sigma_v^2}\right]_{i,j}^H =$$

$$\frac{1}{2} \, \text{tr} \left\{ C_y^{-1} \sigma_{x_{i_{N_t}}}^2 \, \lambda_{i_{N_t}} \, \frac{\partial \lambda_{i_{N_t}}^H}{\partial h_i^*} C_y^{-1} \lambda_j \lambda_j^H \right\}, \quad 1 \leq i \leq N_t N_r N \; 1 \leq$$

$$j \leq N_t \quad (18)$$

$$\left[J_{h\sigma_v^2} \right]_i = \left[J_{hh^* \sigma_v^2} \right]_{i,j}^H =$$

$$\frac{1}{2} tr \left\{ C^{-1} \sigma_{x_{i_{N_t}}}^2 \, \lambda_{iN_t} \, \frac{\partial \lambda_{i_{N_t}}^H}{\partial h_i^*} C_y^{-1} \right\}, \; 1 \leq i \leq N_t N_r N$$

$$\left[J_{\sigma_x^2 \sigma_v^2} \right]_i = \frac{1}{4} tr \left\{ C_y^{-1} \lambda_i \lambda_i^H C_y^{-1} \right\}, \; 1 \leq i \leq N_t \quad (19)$$

Once the total FIM $J_{\theta\theta}$ is obtained by the summation of the two FIMs given by equations (13) and (18), it is inverted to obtain the CRB matrix. Then, the top-left N $N_t N_r \times$ N $N_t N_r$ subblock of the CRB matrix (referred to as h-block) is extracted the deduce the CRB_{SB}^{CG} for the channel parameter vector.

V. RESULTS

This chapter included the discussion about the results obtained from the simulation of semi-blind channel estimation for MIMO-OFDM system and their comparisons with block type lattice type and comb type pilot arrangements.

A. Simulation parameters

Tool	MATLAB
No. of subcarrier	16,64,128
Channel	AWGN
Data Size	Min-1000 and max- 10000bits
Modulation	QPSK,BPSK4
FFT length	256 and 512
N_p(No. of pilots)	32
$N_T = N_R$	4
Carrier freq.	5 Ghz

B. Comparison of CRBs Vs SNR

Fig.3 compares the normalized CRB versus SNR. The CRB curves confirm that the CRBs of semi-blind channel estimation are lower than the CRB when only pilots are exploited (CRB_{OP}). Note that CRB_{SB}^{NCG}, gives better results than the while the BPSK and QPSK cases provide the best CRB_{SB}^{CG}, results.
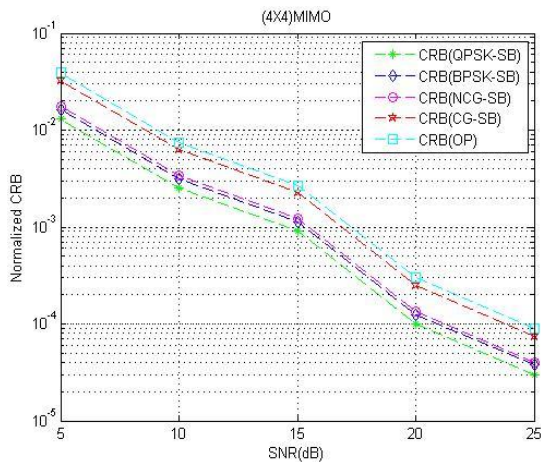

Fig 2: CRB Vs SNR

C. Comparison between CRB and the number of deleted pilot samples

This section analyzes the limit bounds of the channel estimation performance when comb-type pilots arrangement is used. The number of pilot samples per OFDM symbol is Kp = 8, Kd = 56 for data, Ns = 40, while the other simulation parameters and the channel model) are given in Table 6.1.
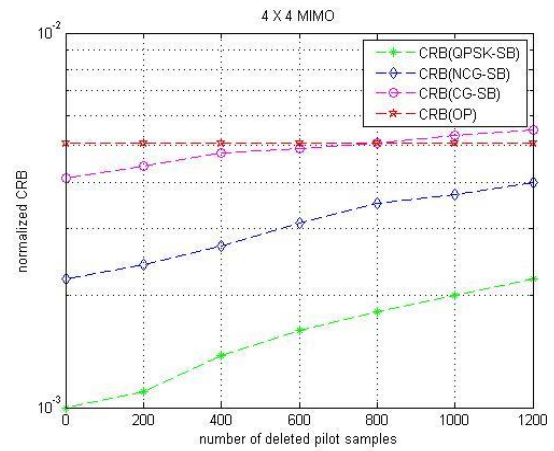

Fig 3:CRB Vs deleted symbols

CONCLUSION

This paper has focused on the theoretical performance limit of the semi-blind channel estimation in MIMO-OFDM and large MIMO-OFDM systems. Analytical derivations of the channel estimation CRBs have been provided for different data models and for different pilot design patterns (i.e. block-type, lattice-type and comb-type pilot arrangement).In particular, the previous analytical study includes new CRB derivations for the Non-Circular Gaussian and the BPSK/QPSK data model cases. For the latter, a realistic CRB approximation has been given to bypass the high complexity of the exact BPSK/QPSK CRB computation. Another contribution of this paper consists of an effective computational technique to deal with the huge- size matrix manipulation needed for the CRB calculation in the large size MIMO scenario. Finally, based on the previous CRB derivation, thorough investigations of the pilot reduction potential of the semi-blind channel method has been conducted in the contexts of IEEE 802.11n MIMO-OFDM and large MIMO-OFDM, respectively.

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