

Modelling the Data of Traffic Accidents with Sinusoids Curve Fitting

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Abstract: In this study, the number of deaths and injured in traffic accidents happened in Turkey between January 2012 - June 2018 were modelled with trigonometric functions. A sinusoidal curve was attempted on the data related to the number of deaths and injured caused by traffic accidents with a minimum error sum of squares. The parameter estimates of the model formed by using trigonometric functions and Fourier series were found to be significant ($P < 0.001$). Determination Coefficients (R^2) was found as 0.913, 0.855 and 0.922 respectively. Durbin-Watson statistics were found as 1.695, 1.852 and 2.243 respectively. It was found that the number of accidents, deaths and injured were periodically higher in June, July, August and September and lower in December, January, February and March according to the analysis performed.

Keywords: Sinuzoids curve, Fourier series, traffic accident, killed, injured.

I. INTRODUCTION

The periodical functions used to examine oscillation and vibration movements are expressed with Fourier series (Altın, 2011). Sinusoidal curve fitting could be implemented in various fields having periodical incidents, as well as traffic as we encounter traffic incidents with periodically progressing accidents, deaths and injured.

As freight shipment and passenger transportation is mostly performed by land transportation in Turkey and the lack of a safe traffic environment cause an increase in traffic accidents. As a result of these accidents; deaths, injured, disabilities and major economic losses occur. Detailed statistical information is required to determine the precautions to be taken and investments to be made to ensure the traffic safety in our country.

Traffic accidents pointed out as one of the most significant problems in Turkey have shown a continuous rise. While 1.182.419 traffic accidents happened in 2016, 7300 people died and 303.812 were injured in these accidents. (TIS, 2016).

Anticipatory accident estimations should be known and modelled to determine the plans and policies related to the road safety (Akgüngör and Doğan, 2008). Time series method has also been used in studies related to traffic accident estimations (Gandhi and Hu, 1995; Van den Bossche et al, 2004; McLeod and Vingilis, 2008; Quddus, 2008).

Some researches about sinusoidal curve fitting were also available. Fridstrøm and Ingebrigtsen (1991); (Oppe, 1991; Chang and Graham, 1993; Keeler, 1994; Van den Bossche et al., 2004; Quddus, 2008) have studied on monthly traffic accident estimation models. Ögüt (1998) aimed to model daily traffic flows by using spectral analysis or Fourier analysis method in his study. In their studies, Çetin et al. (1999) determined the spatial alteration structures of long term annual monthly rainfall observations in Eastern Mediterranean Region with the geostatistical method. In their study named "Determination of bridge oscillation and vibrations with real

time kinematic GPS" Mekik et al. (2005) modelled the changes in oscillation and vibration throughout the periods when traffic and heavy vehicles use the road more frequently and more infrequently. In the study of Abbak (2005), sea level observations were analyzed with spectral analysis method using smallest squares. Yurtçu and İçağa (2005), calculated periodical components of the data through the observations of water level of the well, flow, rainfall and vaporization from 5 wells, 4 rainfall, 6 flow and 4 vaporization surveillance station located at the Akarçay basin. According to the periodogram analysis results aimed to examine whether there is any intra-year periodical behavior, (2) periods were found in Seyitler (11001), Selevir (11004) rainfall observation data; and (1) period was found in Afyon (1034), Çay (11021) rainfall observation data. Two periods throughout the year have shown a change in four months in other words the seasonal change.

The aim of this study is to perform a curve fitting with the help of sinusoid functions (trigonometric functions) and implement them on traffic data.

II. MATERIAL AND METHOD

Material

The material of this study is composed of the monthly traffic data for the years 2012-2018 received from the Directorate General of Security, Traffic Services Directorate, Traffic Newsletter.

Method

Let $f(x)$ be defined in the interval $(-L, L)$ and outside of this interval by $f(x+2L)=f(x)$, assume that $f(x)$ has the period $2L$. The Fourier series (Fourier expansion) corresponding to $f(x)$ is given by

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right) \quad (1)$$

where the Fourier coefficients a_n and b_n are

$$\begin{cases} a_n = \frac{1}{L} \int_{-L}^L f(x) \cos \frac{n\pi x}{L} dx \\ b_n = \frac{1}{L} \int_{-L}^L f(x) \sin \frac{n\pi x}{L} dx \end{cases} \quad n = 0, 1, 2, \dots \quad (2)$$

To determine a_0 in (1), it be used (2) with $n=0$. To illustrate, from (2) it be seen that $a_0 = \frac{1}{L} \int_{-L}^L f(x) dx$. Note that the constant term in (1) is equal to $a_0 = \frac{1}{2L} \int_{-L}^L f(x) dx$, which is the mean of $f(x)$ over a period. If $L=\pi$, the series (1) and the coefficients (2) are particularly simple. The function in this case has the period 2π (Spiegel, 1963).

Considering these properties, any $f(x)$ function,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \quad (3)$$

can be expansion to the Fourier series. This series is called the trigonometric series(Khuri, 2003).

Required conditions have to be met to be able to approximately predicate any $f(x)$ function with Fourier series. These conditions are; function being monovalent in each continuous point between the desired range, being finite and continuous in the desired range, and taking the maximum or minimum values in the desired range (Bayram, 2002). Under these conditions, Fourier series converge on $f(x)$ where function is continuous. On the points with discontinuity, it converges on the arithmetical average of the right and left limits of the series function (Buttkus, 2000).

If $f(x)$ function is not continuous and is given as the values in n number of equally spaced separate points, total symbols are used instead of the integral operations specified above. If the values of the y variable show a periodical variation against the increasing values of the x variable, the discontinuous Fourier series will be selected as the mathematical model. The periodical component formed with the effect of the annual period in any parameter of the series is determined with the Equation 4 as follows;

$$y_i = a_0 + \sum_{k=1}^m \left[a_k \cos \left(\frac{2\pi}{T} kx_i \right) + b_k \sin \left(\frac{2\pi}{T} kx_i \right) \right] + e_i \quad (4)$$

Here: y_i : Average value of the parameter, m : Significant harmonic number, a_k, b_k : Fourier coefficients, T : is period (Salas and Yevjevich, 1972). e_i in Equation 4 is also the residuals (error) value of time (Bloomfield, 2000).

$$\theta_i = \frac{2\pi}{T} x_i, \quad i=1,2,\dots,n$$

After the conversion

$$s = \sum_{i=1}^n \left\{ y_i - \left[a_0 + \sum_{k=1}^m (a_k \cos k\theta_i + b_k \sin k\theta_i) \right] \right\}^2 \quad (5)$$

is written down as above (Türker and Can, 1997). When this equation is minimized

$$a_0 = \frac{1}{n} \sum_{i=1}^n y_i a_0 = \frac{1}{n} \sum_{i=1}^n y_i,$$

$$a_k = \frac{2}{n} \sum_{i=1}^n y_i \cos k\theta_i, \quad b_k = \frac{2}{n} \sum_{i=1}^n y_i \sin k\theta_i$$

and

$$b_k = \frac{2}{n} \sum_{i=1}^n y_i \sin k\theta_i, \quad b_k = \frac{2}{n} \sum_{i=1}^n y_i \sin k\theta_i$$

formulas are acquired. Considering $n=1$

$$y_i = a_0 + a_1 \cos \theta_i + b_1 \sin \theta_i + e_i, \quad i=1,2,\dots,n$$

$$s = \sum_{i=1}^n [y_i - a_0 - a_1 \cos \theta_i - b_1 \sin \theta_i + e_i]^2 \quad (6)$$

could be written down. When this signification is minimum and if the partial derivative is taken according to the A_0, A_1 and B_1 parameters:

$$na_0 + a_1 \sum_{i=1}^n \cos \theta_i + b_1 \sum_{i=1}^n \sin \theta_i = \sum_{i=1}^n y_i$$

$$a_0 \sum_{i=1}^n \cos \theta_i + a_1 \sum_{i=1}^n \cos^2 \theta_i + b_1 \sum_{i=1}^n \cos \theta_i \sin \theta_i = \sum_{i=1}^n y_i \cos \theta_i$$

$$a_0 \sum_{i=1}^n \sin \theta_i + a_1 \sum_{i=1}^n \sin \theta_i \cos \theta_i + b_1 \sum_{i=1}^n \sin^2 \theta_i = \sum_{i=1}^n y_i \sin \theta_i$$

Above stated equations are found. These equations are written down as

$$\begin{bmatrix} n & \sum \cos \theta_i & \sum \sin \theta_i \\ \sum \cos \theta_i & \sum \cos^2 \theta_i & \sum \cos \theta_i \sin \theta_i \\ \sum \sin \theta_i & \sum \cos \theta_i \sin \theta_i & \sum \sin^2 \theta_i \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum y_i \cos \theta_i \\ \sum y_i \sin \theta_i \end{bmatrix}$$

in the matrix form. For the sums in coefficients matrix, if the system in the equation is solved as

$$\frac{1}{n} \sum_{i=1}^n \sin \theta_i, \quad \frac{1}{n} \sum_{i=1}^n \cos \theta_i = 0; \quad \frac{1}{n} \sum_{i=1}^n \sin^2 \theta_i = \frac{1}{2}$$

$$\frac{1}{n} \sum_{i=1}^n \cos^2 \theta_i = \frac{1}{2}; \quad \frac{1}{n} \sum_{i=1}^n \sin \theta_i \cos \theta_i = 0$$

$$\begin{bmatrix} a_0 \\ a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} 1/n & 0 & 0 \\ 0 & 2/n & 0 \\ 0 & 0 & 2/n \end{bmatrix} \begin{bmatrix} \sum y_i \\ \sum y_i \cos \theta_i \\ \sum y_i \sin \theta_i \end{bmatrix}$$

will be the result (Türker ve Can, 1997).

Provided that there is both trend and seasonal fluctuations in a series, the regression model to be implemented is as follows:

$$Y_t = a + \sum_{i=1}^m b_i t^i + \sum_{i=1}^{\lfloor s/2 \rfloor} \left[c_i \sin \left(\frac{2\pi i t}{s} \right) + d_i \cos \left(\frac{2\pi i t}{s} \right) \right] + e_t \quad (7)$$

Here, m is the polynomial degree of the series. Here, the trend constituent of the series is expressed with the sum, $\sum_{i=1}^m b_i t^i$. In

this equation, s is the period and the $\lfloor s/2 \rfloor$ is the integer part of the half of the period. Here j index is the seasonal component of the total series. According to these information, sinus and cosine function pair is called harmonic.

In regression analysis implemented to the seasonal series, the coefficients are found with the following formula.

$$\beta = (X'X)^{-1} X'Y$$

If there is a linear trend and a harmonic in the additive model, the equation is as follows:

$$\beta = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}_{4 \times 1} \text{ and}$$

$$X = \begin{bmatrix} 1 & 1 & \sin(2\pi/s) & \cos(2\pi/s) \\ 1 & 2 & \sin(4\pi/s) & \cos(4\pi/s) \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 1 & T & \sin(2\pi T/s) & \cos(2\pi T/s) \end{bmatrix}_{T \times 4}$$

(Kadilar, 2009).

Sinusoids curve fitting

Annual cycle may be expected to represented in the form

$$S_t = \mu + R \cos 2\pi(ft + \phi) \quad (8)$$

where the frequency is $f = \frac{1}{12}$ cycles per month. The data

$\{x_0, x_1, \dots, x_{n-1}\}$ will be modeled as $X_t = S_t + e_t$ where e_t is the residual at time t. The term μ is an added constant(Bloomfield 2000).

The three-parameter "sinusoid plus constant" model

$$S(\mu, A, B) = \sum_{t=0}^{n-1} (X_t - A \cos 2\pi ft - B \sin 2\pi ft)^2$$

with respect to μ , A and B are

$$\frac{\partial T}{\partial \mu} = -2 \sum_{t=0}^{n-1} (X_t - \mu - A \cos 2\pi ft - B \sin 2\pi ft)$$

$$\frac{\partial T}{\partial A} = -2 \sum_{t=0}^{n-1} \cos 2\pi ft (X_t - \mu - A \cos 2\pi ft - B \sin 2\pi ft)$$

$$\frac{\partial T}{\partial B} = -2 \sum_{t=0}^{n-1} \sin 2\pi ft (X_t - \mu - A \cos 2\pi ft - B \sin 2\pi ft)$$

respectively. These expressions using the solve them for the least squares estimates $\hat{\mu}$, \hat{A} and \hat{B} of μ , A and B, respectively (Bloomfield 2000).

III. RESULTS

Traffic accidents by months in Turkey between January 2012 – June 2018 (accidents resulted in killed and injured) have been shown respectively in Figure 1, Figure 2 and Figure 3.

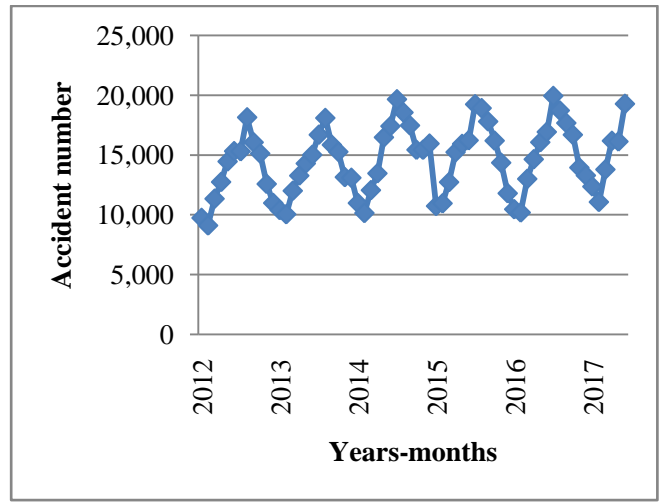


Figure1.Number of traffic accidents in Turkey in January 2012 – June 2018 period.

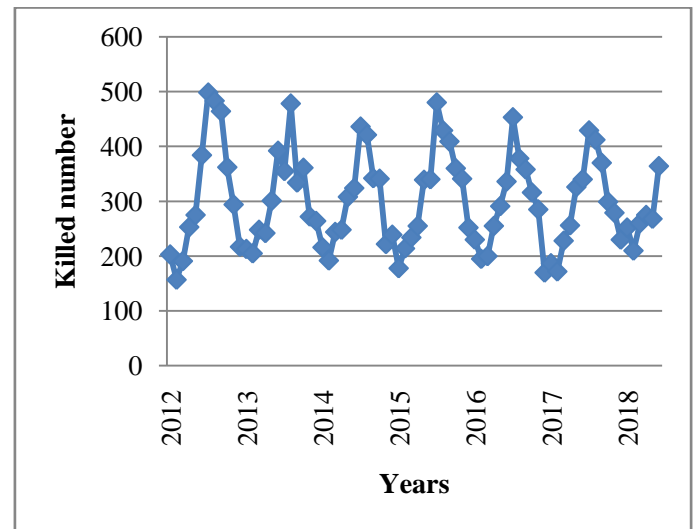


Figure 2.Number of killed in traffic accidents in Turkey in January 2012 – June 2018 period

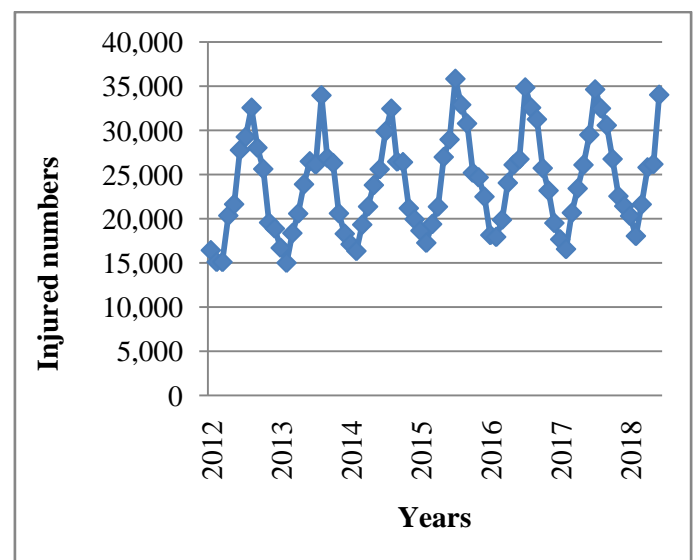


Figure 3.Number of injured in traffic accidents in Turkey in January 2012 – June 2018 period

As seen in Figure 1, Figure 2 and Figure 3, there is a seasonal trend in the number of accidents, deaths and injured data graphics. It is seen as if the traffic accidents and the number of deaths and injured have a periodical behavior based on the seasons. The period of these series is 12. As the periodical

functions used to indicate periodical incidents are shown with trigonometric series (Fourier series) (Altun, 2011), trigonometric curve (sinusoid curve) fitting was performed. Generally, less traffic accidents occur in winter (January, February and March) whereas there are more accidents in summertime (July, August and September). After the summer, the accidents happen most frequently in May and June. The same trend is also applicable for the accidents continue periodically for many years and for the period of the study. A sinusoid curve that includes the appropriate sinus and cosinus fluctuations could be formed for this data. The same case is also applicable for the number of deaths and injured.

Sinusoid model for the number of traffic accidents between January 2012 - June 2018 is applied. The period of the series is T=12.

$$\theta_i = \frac{2\pi}{12} t_i = \frac{\pi}{6} t_i \quad i=1,2,\dots,12$$

conversion is performed. By using the values in Table 2, the coefficients in Equation 8 are obtained.

Table 1. sinus and cosinus values formed according to the t_i values

t_i	0	1	2	3	4	5	6	7	8	9	10	11
θ_i	0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π	$7\pi/6$	$4\pi/3$	$3\pi/2$	$5\pi/3$	$11\pi/6$
Sin θ_i	0	0.500	0.866	1	0.866	0.500	0	-0.500	-0.866	-1	-0.866	-0.500
Cos θ_i	1	0.866	0.500	0	-0.500	-0.866	-1	-0.866	-0.500	0	0.500	0.866

Regarding the number of accidents in traffic accidents, the number of accidents is the dependent variable and the variables that include sinus and cosinus terms as well as the t variable indicating the time are the independent variables. According to the regression analysis results in Table 2, a=12645.865, b=44.342, c=-2765.201, d=-2494.879 coefficients that belong to the number of accidents series are found. According to the Equation 8,

$$y_i = 12645.865 + 44.342 t - 2765.201 \sin\theta_i - 2494.679 \cos\theta_i + \varepsilon_t$$

sinusoid curve is found. Moreover, this series is also similar to the Fourier Series Analysis model as the sinus and cosinus terms forming this model are the terms of the Fourier series. Parameter estimations related to the regression model are found significant (P<0.001). The general significance test of the model was also found significant (F=258.037 and P<0.01). Durbin-Watson coefficient is DW=1.695 and there is no autocorrelation problem.

Table2.The results of traffic accidents regression analysis

Variables	Coefficients	Standard error	t statistics	p
Constant	12645.865	204.741	61.765	0.001
b	44.342	4.5	9.853	0.001
c (sin)	-2765.201	143.625	-19.253	0.001
d (cos)	-2494.879	143.322	-17.408	0.001

F=258.037 ve P<0.001'dir. DW=1.695 ve R²=0.913

Regarding the number of deaths in traffic accidents, the number of deaths is the dependent variable and the variables that include sinus and cosinus terms as well as the t variable are the independent variables. The model related to the number of deaths according to the regression analysis results shown in Table 3 is as follows;

$$y_i = 311.176 - 0.186 t - 89.577 \sin\theta_i - 68.722 \cos\theta_i + \varepsilon_t$$

In the parameter estimations of the regression model, coefficients of the terms with sinus and cosinus were found significant (P<0.001). The general significance test of the model was also found significant (F=258.037 and P<0.01).

Durbin-Watson coefficient is DW=1.852 and there is no autocorrelation problem.

Table3.The results of the number of deaths regression analysis

Variables	Coefficients	Standard error	t statistics	p
Constant	311.176	7.735	40.228	0.001
b	-0.186	0.170	-1.092	0.278
c(sin θ_i)	-89.577	5.426	-16.508	0.001
d(cos θ_i)	-68.722	5.415	-12.692	0.001

F=258.037 ve P<0.001'dir. DW=1.852 ve R²=0.855

Regarding the number of the injured, the number of injured is the dependent variable and the variables that include sinus and cosinus terms as well as the t variable are the independent variables. The model related to the number of injured, according to the regression analysis results in Table 4 is as follows;

$$y_i = 22007.211 - 56.655 t - 5397.639 \sin\theta_i - 4838.973 \cos\theta_i + \varepsilon_t$$

In the parameter estimations of the regression model, coefficients of the terms with sinus and cosinus were found significant (P<0.001). The general relevance test of the model was also found significant (F=291.587 and P<0.01). Durbin-Watson coefficient is DW=2.243 and there is no autocorrelation problem.

Table 4.Results of the regression analysis of the number of injured

Variables	Coefficients	Standard error	t statistics	p
Constant	22007.211	361.042	60.955	0.001
b	56.655	7.936	7.139	0.001
c(sin θ_i)	-5397.639	253.269	-21.312	0.001
d(cos θ_i)	-4838.973	252.734	-19.147	0.001

F=291.587 ve P<0.001'dir. DW=2.243 ve R²=0.922

According to the models formed for the number of accidents, deaths and the injured, the common graphic of the actual and estimate values of 2012:1-2018:6 is shown at Figure 4-Figure 6. In Figure 4, Figure 5 and Figure 6, there is a correlation between the actual values and estimated values of series of the

accident, death and injured respectively. According to the estimation results, it is found that the period with the highest number of accidents, deaths and injured is July, August and September and the lowest is the period of January, February and March.

CONCLUSION

In this study, data analysis was performed through sinusoids curve. The number of traffic accidents, deaths and injured between January 2012 - June 2018 were examined. As the number of accidents, deaths and injured of the stated period showed periodic behavior, they are modelled with sinusoid curve that demonstrates sinus and cosinus fluctuations. The number of accidents, deaths and injured were estimated by means of obtained models. According to the estimation results, the number of accidents, deaths and injured are estimated to be lower periodically in winter and higher in summer. Plans and programs could be carried out in order to take precautions, especially in summer time to ensure traffic safety and reduce the number of accidents as well as the number of deaths and injured.

References

- [1] Abbak, R. A. 2005. Deniz Düzeyi Gözlemlerinin En Küçük Kareler Yöntemiyle Spektral Analizi. Yüksek Lisans Tezi, Selçuk Üniversitesi Fen Bilimleri Enstitüsü Jeodezi ve Fotogrametri Mühendisliği Anabilim Dalı, Konya.
- [2] Akgüngör, A.P., Doğan, E. 2008. Smeedve Andreassen kaza modellerinin Türkiye uygulaması: Farklı senaryo analizleri. Gazi Üniversitesi Mühendislik Mimarlık Fakültesi Dergisi, 23(4):821-827.
- [3] Altın, A. 2011. Fourier Analizi, Gazi Kitapevi, Ankara.
- [4] Anonymous, 2015. Emniyet Genel Müdürlüğü. Trafik Hizmetleri Başkanlığı. Genel Kaza İstatistikleri. <http://www.trafik.gov.tr/Sayfalar/Istatistikler/Genel-Kaza.aspx> (Accessed to:25.05.2015).
- [5] Bayram, M. 2002. Fen ve Mühendisler İçin Nümerik Analiz. Aktif Yayınevi, İstanbul, 374- 377.
- [6] Bloomfield, P. 2000. Fourier analysis of Time Series An Introduction. John Wiley Sons, Inc, 9-14.
- [7] Butkus, B. 2000. Spectral Analysis and Filter Theory in Applied Geophysics. Springer.
- [8] Chang, B.H.,Graham, J.D. 1993. A new method for making interstate comparisons of highway fatality rates. Accident Analysis and Prevention 25(1):85-90.
- [9] EGM, 2018. Emniyet Genel Müdürlüğü Trafik Hizmetleri Başkanlığı Trafik İstatistik Bülteni Haziran 2018, Ankara.
- [10] Fridström, L., Ingebrigtsen, S. 1991. An aggregate accident model based on pooled, regional time series data. Accident Analysis and Prevention 23(5):363-378.
- [11] Gandhi, U. N., Hu, S. J. 1995. Data-based approach in modeling automobile crash. International Journal of Impact Engineering, 16(1):95-118.
- [12] Kadılar, C. 2009. SPSS Uygulamalı Zaman Serileri Analizine Giriş. Bizim Büro Basımevi, Ankara, 300 p.
- [13] Keeler, T.E. 1994. Highway safety, economic behavior and driving enforcement. The American Economic Review, 84(3):684-693.
- [14] Khuri, A. I. 2003. Advanced Calculus with Applications in Statistics. John Wiley Sons, Inc., Canada.
- [15] McLeod, A. I., Vingilis, E. R. 2008. Power computations in time series analyses for traffic safety interventions. Accident Analysis and Prevention, 40(3):1244-1248.
- [16] Mekik, Ç., Görmüş, K. S. & Kutoğlu H. 2005. Gerçek Zamanlı Kinematik GPS ile Köprü
- [17] Salınım ve Titreşimlerinin Belirlenmesi, Harita ve Kadastro Mühendisleri Odası, Mühendislik Ölçmeleri STB Komisyonu 2. mühendislik Ölçmeleri Sempozyumu, İTÜ-İstanbul, 161-176.

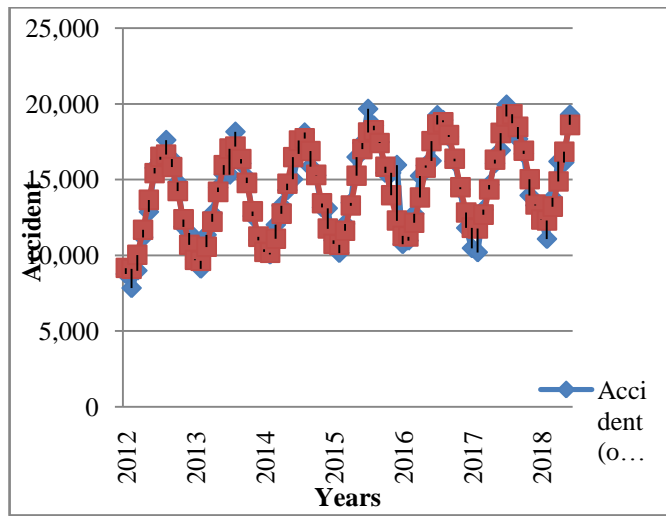


Figure 4. Joint graphic of the original series of the number of accidents and estimation series

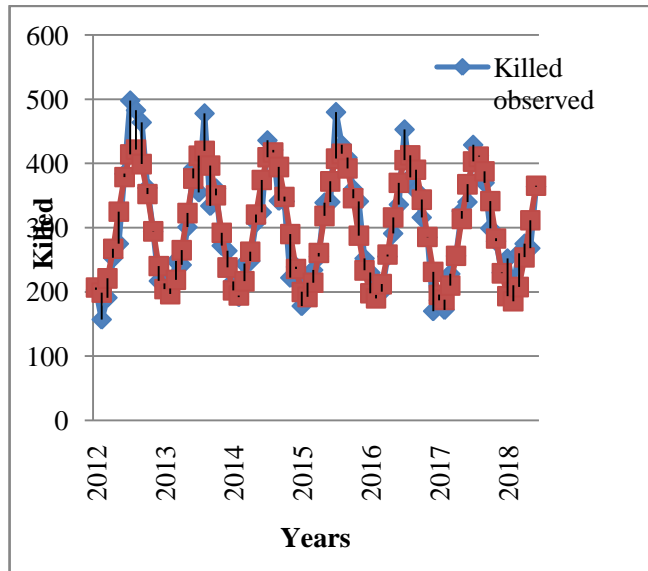


Figure 5. Joint graphic of the original series of the number killed and estimation series

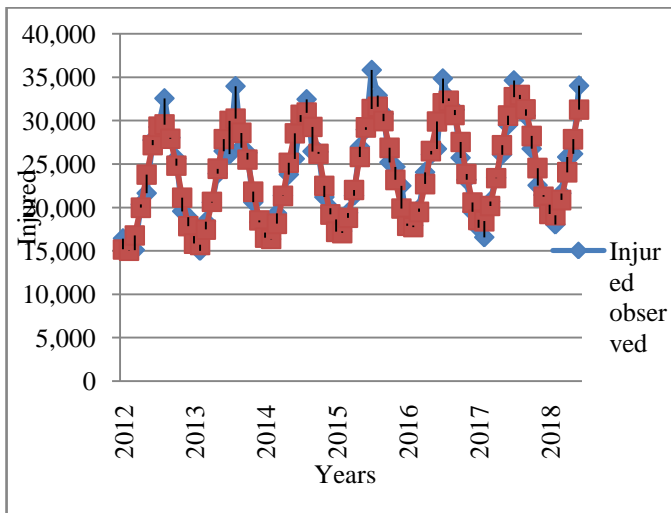


Figure 6. Joint graphic of the original series of the number of injured and estimation series

- [18] Oppe, S. 1991. Development of traffic and traffic safety: global trends and incidental fluctuations. *Accident Analysis and Prevention* 23(5):413–422.
- [19] Öğüt, K. S. 1998. Trafik Akımlarının Spektral Analiz Yöntemi ile Modellenmesi. 4. Ulaştırma Kongresi, Denizli, 1-9.
- [20] Prestly, M. B. 1981. *Spectral Analysis and Time Series*. Academic Press, London, New York.
- [21] Quddus, M. A. 2008. Time series count data models: An empirical application to traffic accidents. *Accident Analysis and Prevention*, 40:1732–1741.
- [22] Salas, J. D., Yevjevich, V. 1972. Stochastic Structure of Water Use Time Series, *Hydrology Papers*, No. 52, Colorado State University, Fort Collins-Colorado, 71.
- [23] Spiegel, M. R. 1963. *Advanced Calculus*. Mc-Graw Hill Inc., New York, 298-300.
- [24] TIS, 2016. Turkey Institute Statistics. General Directorate of Public Security and General Command of Gendarmerie. Number of accidents, persons killed and injured by year. http://www.tuik.gov.tr/PreTablo.do?alt_id=1051
- [25] Türker, E. S., Can, E. 1997. Bilgisayar Uygulamalı Sayısal Analiz Yöntemleri. Değişim Yayınları, Adapazarı, 234-239.
- [26] Van den Bossche, F., Wets, J., Brijs, T. 2004. A regression model with ARIMA errors to investigate the frequency and severity of road traffic accidents. In *Proceedings of the 83rd annual Meeting of the Transportation research Board* (pp. 11–15), Washington, DC, USA, January 2004.
- [27] Yurtcu, S., İçağa, Y. 2005. Akarçay Havzası Yeraltı Suyu Periyodik Davranışının Modellenmesi. *Yapı Teknolojileri Elektronik Dergisi*, 2:21-28.