Analysis of Locational Marginal Pricing Approach for a Deregulated Electricity Market

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Abstract- In a deregulated power industry, estimation of power price and management of congestion is a major issues handled by market members. Modeling of sensible pricing structure of power systems is important to provide financial signals for electrical utilities. In restructured electricity markets, a valuable transmission pricing method is required to compact with transmission issues and to generate exact economic signals to condense the generation cost. It is essential to build up a suitable pricing scheme that can offer the valuable information to market users, such as generation companies, transmission companies and customers. These pricing depends on generator bids, load levels and transmission network constraints. Transmission line constraints can result in deviation in energy prices all over the network. Managing of Congestion is one of the major problem in deregulated electricity pricing with the enhance of complexity to the system. Spot Pricing or Locational Marginal Pricing is a prevailing approach in energy market operation and planning to identify nodal prices and manage congestion in transmission systems. Locational Marginal Pricing (LMP) obtains from the optimal power flow problem gives the economic value of electrical energy at each location. Proposed approach is based on DC optimal power flow model with and without considering of losses. To solve this LMP problem optimization based Quadratic Programming (QP) approach has been implemented. In this paper LMP values with normal condition, congestion condition, and marginal loss condition are studied. IEEE 14 bus and 30 bus system are used as the test system in this paper.

Keywords— *Locational Marginal Pricing (LMP), DC Optimal PowerFlow (DCOPF),Quadratic Programming (QP),Marginal Loss Price,Generation Shift Factor(GSF), Loss Factor*

I. INTRODUCTION

Due to inefficient operation of vertically incorporated power industry it goes to deregulation by separating its operation into multiple services, by means of GENCO's, TRANSCO's and DISCO's. This will lead to the inefficient operation of power system. So the electric power industry has undergone deregulation around the world, a core tenet of which is to build an open-access, unambiguous and fair electricity markets [6]. Due to central operation of transmission and distribution system it will remain in a monopoly mode. Under the deregulated electricity `market environment, transmission networks play a vital role in supporting the transaction between producers and consumers. One drawback of transmission network is overloading. Federal Energy Regulatory Commission (FERC) willing to create non-profit organizations, called Independent System Operator System (ISO) and Regional Transmission Organization (RTO), to organize regional power systems to ensure non-discriminatory transmission services to generation companies (GENCO's) and bilateral transactions. In the restructured power industry open access is provided to the transmission system. Due to Transmission Open Access (TOA) the power flow in the lines reach the power transfer limit and so it will leads to a condition known as congestion [1-2]. The congestion may be caused due to a mixture of reasons, such as transmission line outages, generator outages and change in energy demand. Transmission congestion has impact on the entire system as well as on the individual market participants i.e. sellers and buyers. Without congestion low cost GENCO's are used to meet the load demand but if congestion is present in the transmission network then it prevents the demand to be met by the lowest-priced resources due to mentioned transmission constraints and this leads to the allocation of higher price GENCO's.

There are two types of pricing methods are available in practice for congestion management [11]. They are uniform and non-uniform pricing structure. In this paper congestion is managed by means of Locational Marginal Pricing (LMP) i.e. non-uniform pricing structure. The LMP at a location is defined as the marginal cost to supply an additional MW increment of power at the location without violating any system security limits [13]. This price reflects not only the marginal cost of energy production, but also its delivery. Because of the effects of both transmission losses and transmission system congestions, LMP can vary significantly from one location to another. If the lowest priced electricity is allocated for all Location LMP values at all nodes will be same. If congestion present in the system lowest cost energy cannot reach all location, more expensive generators will allocated to reach out the demand. In this situation LMP values will be differ from one location to another. In pool-based electricity market ISO collects hourly supply and demand bids from Generator Serving Traders (GSTs) on behalf of GENCO's and Load Serving Traders (LSTs) on behalf of pool consumers [6]. ISO determines the generation and demand schedule as well as LMPs based on increased social welfare maximization, subject to system operational and security constraints [9-11]. Mathematically, LMP at any node in the system is the dual variable for the equality constraint at that node [4]. Buyers in the market pays ISO based on their price for dispatched energy. The ISO pays sellers in the market based on their respective prices. The LMP difference between two adjacent buses is the congestion cost which arises when the energy is transferred from one location to the other location. Marginal losses represent incremental changes in system losses due to incremental demand changes. Incremental losses yield additional costs which are referred to as the cost of marginal losses. Thus LMP is the summation of the costs of marginal energy, marginal loss and congestion cost.

LMP can be stated as follows:

LMP = generation marginal cost + congestion cost + marginal loss cost

LMP is obtained from the result of Optimal Power Flow (OPF). Either AC-OPF or DC-OPF is used to determine the LMP [7]. To reduce the complexity in the calculation in this paper DC-OPF is used. In DC-OPF only real power flow is considered [6]. Different types of optimization models are used for LMP calculations like LP and Lagrangian relaxation using karush–kuhn-Tucker conditions. Evolutionary algorithm like genetic algorithm [12] is also used. Among these in this paper QP is used to solve the optimization problem.

A. Types of Bids

Most commonly a generator bid varies with many factors, some of the factors are difficult to model. For simplicity generator bids are assumed to be equal to their incremental costs for perfect competition. There are two bidding models available in practice [12]. They are

(1) Fixed generator bids (related to piecewise-linear heat rates)

(2) Linear bids (related to quadratic heat rates)

In this paper linear bids are used to calculate the generator offer price. Linear bid function is defined as a quadratic function and it is given by the following equation,

$$C_i(PG_i) = a_i + b_i\, PG_i + c_i\, PG_i^2 \text{ (\$/hr)} \qquad (1)$$

Where,

$C_i(PG_i)$ - cost of generating i^{th} unit

a_i - no-load cost

b_i - linear cost coefficient

c_i - quadratic cost coefficient of unit i.

These coefficients are given by the generator manufacturer.

B. Day-Ahead and Real-Time Energy Markets

Restructured power market consists of different types of market. An energy market is a place where the financial trading of electricity takes place. It naturally consists of a day-ahead market and real-time market, while the ancillary service markets are able to provide services such as synchronized reserve, regulation and reliable operation of transmission system. The day-ahead market is a type of forward market and runs on the day before the functioning day [1-2] .Generation offers, demand bids, and bilateral transactions are accepted by the Day-Ahead market in the regulated market timeline.

Normally, LMP generated by the day-ahead market is called “ex-ante LMP”, because the LMP is calculated before the energy a transaction happens. In the real-time market, “post-LMP” calculation will be performed as like that of “ex-ante LMP”. Basically “ex-ante LMP” will be same as that of “post-LMP” if the forecasted load reflects the actual load in the real time market. In this paper Day-ahead market and “ex-ante LMP” is considered. LMP in the deregulated market depends on various factors such as low cost generator outage, transmission line outage, transmission line limits, load changes, demand bids and generation offers of consumers. In this paper we mainly focus on transmission line limit [4] and generation limit [5] as a constraint.

The paper is structured as follows: Section 2 provides the existing transmission pricing method. Section 3 provides the mathematical formulation of LMP. Section 4 presents the DC-OPF problem formations. Section 5 provides the Quadratic Programming method. Section 6 provides the results and analysis. Section 7 describes conclusion.

II. EXISTING TRANSMISSION PRICING METHOD

Transmission pricing offer global access for all participants in the market. To recover the costs of transmission network and encourage market investment in transmission an understandable price structure is necessary. In this section various pricing methods and their calculations are discussed.

A. Postage-Stamp Rate Method

Postage-stamp rate scheme is conventionally used by electric utilities to allot the permanent transmission price between the users of firm transmission service. This method does not need power flow calculations and is independent of the transmission distance and system arrangement. This transmission pricing method allocates transmission charges based on the amount of the transacted power.

B. Contract Path Method

Contract path method also does not required power flow calculation. In this method contract path is a corporeal transmission pathway among two transmission users that disregards the fact that electrons follow corporeal paths that may differ dramatically from contract paths. Following to the specification of contract paths, transmission prices will then be assigned using a postage-stamp rate, which is determined either individually for each of the transmission systems or on the average for the entire grid.

C. MW-Mile Method

The MW-Mile Method is also called as line-by-line method since it considers, in its calculations, changes in MW transmission flows and transmission line lengths in miles. The method calculates charges associated with each wheeling transaction based on the transmission capacity use as a function of the magnitude of transacted power, the path followed by transacted power, and the distance traveled by transacted power. The MW-mile method is also used in identifying transmission paths for a power transaction. This method requires dc power flow calculations. The MW-mile method is the first pricing strategy proposed for the recovery of fixed transmission costs based on the actual use of transmission network.

Total transmission capacity cost is calculated as follows:

$$TC_t = \text{TC} * \frac{\sum_{k\in K} c_k L_k MW_{t,k}}{\sum_{t\in T}\sum_{k\in K} c_k L_k MW_{t,k}} \qquad (2)$$

Where,

TC_t - cost allocated to transaction t

TC	-	total cost of all lines in $
L_k	-	length of line k in mile
c_k	-	cost per MW per unit length of line k
MW_k	-	flow in line k, due to transaction t
T	-	set of transactions
K	-	set of lines

III. MATHEMATICAL FORMULATION OF LMP

The main objective of this problem is minimization of total cost subjected to energy balance constraint and transmission constraint. Power flow is obtained by DCOPF model with and without considering of losses. In this OPF reactive power is ignored and the voltage magnitudes are assumed to be unity [7]. Earlier studies of LMP calculations with the DCOPF ignore the line losses. Thus, the energy price and the congestion price follow a perfect linear model with a zero loss price. However, challenges arise if losses need to be considered to calculate the marginal loss component in the LMP, especially considering the significance of marginal loss which may be up to 20% different among the different zones in the New York Control Area, based on actual data. The primary challenge of the loss model lies in that the conventional, lossless DC model represents a linear network, but lacks the capability to calculate marginal loss pricing, an important component in the LMP methodology.

Objective function is given by

$$\text{Min} \sum_{i=1}^{n} C_i P_{gi} \tag{3}$$

Subject to

$$\sum_{i=1}^{n} P_{gi} = \sum_{i=1}^{n} P_{di} + Ploss \tag{4}$$

Generation limit constraint is given by

$$Pgi^{min} \leq Pgi \leq Pgi^{max} \tag{5}$$

Transmission line limit is given by

$$lf^{min} \leq lf \leq lf^{max} \tag{6}$$

Where,

i	-	Generator index
n	-	Number of generators
j	-	Line index
C_i	-	Cost of i^{th} generator unit
Pg_i	-	Generation of i^{th} generator unit
Pg_i^{min}	-	Minimum limit of generating unit
Pg_i^{min}	-	Maximum limit of generating unit
Pd_i	-	Demand of i^{th} unit
lf_i^{min}	-	minimum limit of line flow
lf_i^{max}	-	maximum limit of line flow

A. *Generation Shift Factor (GSF)*

Generation shift factor is the ratio of change in power flow of line ‘k’ to change in injection of power at buses ‘i’. GSF coefficient can be computed as,

$$GSF_{k-i} = (B_{a,i}^{-1} - B_{b,i}^{-1})/X_k \tag{7}$$

Where,

B^{-1} = inverse of B (the imaginary part of Y bus matrix)
X_k = reactance of line k
a = sending bus of line k
b = receiving bus of line k

B. *Loss Factor and Delivery Factor*

The key to considering the marginal loss price is the marginal loss factor, or loss factor (LF) for simplicity, and the marginal delivery factor, or delivery factor (DF). Mathematically, it can be written as,

$$\text{DF}_i = 1 - \text{LF}_i = 1 - \partial\text{Ploss}/\partial\text{Pi} \tag{8}$$

Where,

DFi = marginal delivery factor at bus *i*;

LFi = marginal loss factor at bus *i*;

PLoss = total loss of the system;

Pi = *Gi* - *Di* =net injection at bus *i*.

The loss factor and delivery factor can be calculated as follows. Based on the definition of loss factor, we have

$$\partial\text{Ploss}/\partial\text{Pi} = \partial/\partial\text{Pi} \left(\sum_{k=1}^{M} Fk^2 * Rk\right) \tag{9}$$

$$Fk = \sum_{i=1}^{N} GSF_{k-j} * Pj \tag{10}$$

Where,

Fk= line flow at line *k*; *Rk* = resistance at line *k*.

In a linear DC network, a line flow can be viewed as the aggregation of the contribution from all power sources (generation as positive source and load as negative source), based on the superposition theorem. Interestingly, the loss factor at a bus may be positive or negative. When it is positive, it implies that an increase of injection at the bus may increase the total system loss. If it is negative, it implies that an increase of injection at the bus may reduce the total loss.

C. *Total LMP Formulation*

In LMP based electricity markets, system losses have significant impact on the economics of power system operation. So system losses have to be taken into account for obtaining more accurate LMP’s. In this model it is assumed that total system loss is supplied by slack bus generator.LMP loss calculation is given by,

$$LMP\ loss = \lambda * (\text{DFB}-1) \tag{11}$$

$$LMPe = \lambda \tag{12}$$

$$LMPc = \sum_{i=1}^{M} GSF_{k-i} * \mu \tag{13}$$

Where,

LMP loss = Marginal Loss Cost at bus B

LMPe = Marginal Energy Cost

LMPc = Marginal Congestion Cost

Therefore the LMP formulation from equations (11- 13) can be given as,

$$Total\ LMP = LMPe + LMPc + LMPloss \quad (14)$$

IV. FORMATION OF DC-OPF

In AC network real and reactive power transmitted from the generating unit to load centre. Direct Current Optimal Power Flow gives active Power Flow in AC network. This DCOPF is does not have convergence problem i.e. non iterative. From the accuracy level AC-OPF is better than DC-OPF.

Power injection at a node and voltage angles are the important variables for DC-OPF. Active power injection at a bus P_i is given by the Equation (15).

$$P_i = \sum_{j=1}^{n} B_{ij}\,(\theta_i - \theta_j) \quad (15)$$

Where,

Bij – Reactance between bus i and bus j

Power flow on the transmission line is given by the equation (16).

$$P_{Li} = \frac{1}{X_{Li}}(\theta_s - \theta_r) \quad (16)$$

Where,

X_{Li} - Reactance of line i.

DC-OPF equations and power flow in the branch relationship is represented by the Equation (17) & (18).

$$\Theta = [B]^{-1}\, P \quad (17)$$

$$P_L = (b \times A)\, \Theta \quad (18)$$

Where,

P – N x 1 vector of bus active power injection for buses 1,..,N.
B – N x N admittance matrix with R=0.
ϴ – N x 1 vector of bus voltage angle for buses 1,.....,N.
PL – M x 1 vector of branch flows.
M - Number of branches.
b – M x M vector diagonal susceptance matrix.
A – M x N bus – branch incidence matrix. Starting and ending bus elements are 1 and -1 respectively. Otherwise 0.

Earlier studies of LMP calculations with the DCOPF ignore the line losses. Thus, the energy price and the congestion price follow a perfect linear model with a zero loss price. However, challenges arise if losses need to be considered to calculate the marginal loss component in the LMP, especially considering the significance of marginal loss which may be up to 20% different among the different zones in the New York Control Area, based on actual data.

V. QUADRATIC PROGRAMMING

Quadratic programming is a mathematical model to accomplish the finest outcome. This is one of the optimization techniques. It consists of quadratic objective function, subject to equality and inequality conditions in linear form. In the DCOPF with losses model optimization problem is formed as a Quadratic Programming problem. The method creates a sequence of quadratic programming problems that converge to the optimal solution of the original linear problem. Comparing with the older algorithm which uses an augmented Lagrangian, the method has advantages in terms of CPU time and robustness.

Quadratic Programming based optimization is involved in power systems for maintaining a desired voltage profile, maximizing power flow and minimizing generation cost. Here minimization is considered as maximization can be determined by changing the sign of the objective function. Further, the quadratic functions are characterized by the matrices and vectors.

Solving procedure for optimal power flow with Quadratic Programming approach using QP solver is explained in the Following algorithm.

Step1: Formation of quadratic objective function with linear equality and inequality constraint.

Step2: Read the initial values for line and generator data. Also read the generator and line limits.

Step3: Initialize the solution vector X.

Step4:Formation of node–arc incidence matrix to the system.

Step5: Formation of B' matrix.

Step6: Obtain the matrix for power injection and line flow given in the equations (9) & (10) and objective function.

Step7: Solve the obtained matrix by QP solver in the MATLAB.

Step8: Get the LMP value.

VI. RESULTS AND ANALYSIS

The proposed QP method simulation were developed using MATLAB 7.10 software package and the system configuration is Intel Core i3-2328M Processor with 2.20 GHz speed and 2 GB RAM. For simulation work two test systems IEEE 14 bus and IEEE 30 bus system considered as case study. Line and generator data used for the simulation work. Generator offer price is calculated by the quadratic bid function given in Equation (1). For converting the $ into Indian rupee in these paper by simply assuming 1$ equal to 60 rupees.

Following three cases are considered for the analysis.

Case 1 LMP values under normal condition

Case 2 LMP values when congestion occurred

Case 3 LMP values when losses occurred

A. *Case Study – IEEE 14 Bus System*

IEEE 14 bus system consists of 20 lines and 2 generators. Line and generator data's are used for the simulation work..

Three test cases for LMP values calculation are analyzed in this model [14]. Generator data for IEEE 14 bus system is given in the Table 1.

TABLE 1 GENERATOR DATA FOR IEEE 14 BUS SYSTEMS

Generator No	P_i min (MW)	Pi max (MW)	a_i	b_i	c_i
G1	10	20	0.005	2.456	105
G2	20	180	0.005	3.510	44.1

Case 1: LMP is calculated using DCOPF without loss for the IEEE 14 bus system is calculated and presented in the Table 2.

TABLE 2 LMP VALUES UNDER NORMAL CONDITION

Bus. No	LMP ($/MWhr)	Bus. No	LMP ($/MWhr)
1	107.45	8	107.45
2	107.45	9	107.45
3	107.45	10	107.45
4	107.45	11	107.45
5	107.45	12	107.45
6	107.45	13	107.45
7	107.45	14	107.45

From the Table 2, it can be inferred that the LMP does not varies when there is infinite transmission capacity.

Case 2: LMP is calculated using DC OPF without loss for the IEEE 14 bus system, with congestion is created by reducing the line 5 power flow upper limit from 50 MW to 0.772 MW.

TABLE 3 LMP VALUES WHEN CONGESTION OCCURRED

Bus. No	LMP ($/MWhr)	Bus. No	LMP ($/MWhr)
1	107.45	8	171.33
2	47.615	9	190.28
3	94.670	10	195.71
4	135.62	11	228.50
5	33.067	12	256.75
6	262.46	13	252.30
7	171.33	14	217.39

From the Table 3, it can be inferred that the LMP values varies with transmission congestion when any one of the transmission line gets overloading.

Case 3: LMP is calculated using DC OPF with considering of loss for the IEEE 14 bus system is presented in the Table 4.

TABLE 4 LMP VALUES WHEN LOSSES OCCURRED

Bus. No	LMP ($/MWhr)	Bus. No	LMP ($/MWhr)
1	0.880	8	0.000
2	0.069	9	-0.111
3	-0.356	10	-0.034
4	-0.181	11	-0.013
5	-0.028	12	-0.023
6	-0.042	13	-0.051
7	0.000	14	-0.056

From the Table 4, it can be inferred that LMP value is varied depends on any overloading transmission line condition**.**

B. Case Study- IEEE 30 Bus Systems

IEEE 30 bus system consists of 41 lines and 6 generators. Line and generator data's are used for the simulation work. Generator Data consist of maximum and minimum value of generation and cost coefficient values. Three test cases for LMP values calculation are analyzed in this model [14]. Generator data for IEEE 30 bus system is given in Table 5.

TABLE 5 GENERATOR DATA FOR IEEE 30 BUS SYSTEMS

Generator No	P_i min (MW)	Pi max (MW)	a_i	b_i	c_i
G1	0	80	0.00375	2.0000	0
G2	0	80	0.01750	1.7500	0
G3	0	55	0.06250	1.0000	0
G4	0	50	0.00834	3.5000	0
G5	0	30	0.02500	3.0000	0
G6	0	40	0.02500	3.0000	0

Case 1: LMP is calculated using DCOPF without loss for the IEEE 30 bus system is calculated and presented in the Table 6.

TABLE 6 LMP VALUES UNDER NORMAL CONDITION

Bus. No	LMP ($/MWhr)	Bus. No	LMP ($/MWhr)
1	200.375	16	200.375
2	200.375	17	200.375
3	200.375	18	200.375
4	200.375	19	200.375
5	200.375	20	200.375
6	200.375	21	200.375
7	200.375	22	200.375
8	200.375	23	200.375
9	200.375	24	200.375
10	200.375	25	200.375
11	200.375	26	200.375
12	200.375	27	200.375
13	200.375	28	200.375
14	200.375	29	200.375
15	200.375	30	200.375

From the Table 6, it can be inferred that the LMP does not varies when there is infinite transmission capacity.

Case 2: LMP is calculated using DC OPF without loss for the IEEE 30 bus system, with congestion is created by reducing the line 5 power flow upper limit from 45 MW to 0.3 MW.

TABLE 7 LMP VALUES WHEN CONGESTION OCCURRED

Bus. No	LMP ($/MWhr)	Bus. No	LMP ($/MWhr)
1	207.3	16	306.2
2	176.7	17	301.2
3	305.6	18	305.1
4	325.8	19	303.2
5	233.3	20	302.0
6	298.8	21	299.3
7	266.4	22	299.4
8	289.9	23	304.8
9	295.8	24	300.4
10	299.0	25	296.8
11	295.8	26	296.8
12	311.3	27	294.6
13	311.3	28	290.3
14	309.5	29	294.6
15	308.2	30	294.6

From the Table 7, it can be inferred that the LMP values varies with transmission congestion when any one of the transmission line gets overloading.

Case 3: LMP is calculated using DC OPF with considering of loss for the IEEE 30 bus system is presented in the Table 8.

TABLE 8 LMP VALUES WHEN LOSSES OCCURRED

Bus. No	LMP ($/MWhr)	Bus. No	LMP ($/MWhr)
1	-0.449	16	0.000
2	-0.155	17	0.000
3	-0.409	18	0.000
4	-0.449	19	0.000
5	-0.244	20	0.000
6	-0.324	21	0.000
7	0.000	22	0.258
8	0.000	23	0.180
9	0.000	24	0.000
10	0.000	25	0.000
11	0.000	26	0.000
12	0.000	27	0.235
13	0.302	28	0.000
14	0.000	29	0.000
15	0.000	30	0.000

From the Table 8, it can be inferred that LMP value is varied depends on any overloading transmission line condition**.**

CONCLUSION

In a lot of restructured energy markets, the Locational Marginal Pricing acts as an important position in recent times. LMP is looks set to be the most popular congestion management technique adopted by electricity markets around the world. To understand the determination of LMP Loss DC Optimal power Flow is carefully analyzed which is the proposed technique in this paper. Constraints like transmission, generation and transmission line outages are used to analyze the market participants about the location value of electricity. LMP also used to maintain the stable operation of transmission system without affect the buyers and sellers in the market. LMP act as a true price signals for adding transmission capacity, generation capacity and future loads. It achieves its unique ambition of better effectiveness of power system operations in the short-term operational time frames by openly addressing the effects related with power transmission above the interconnected grid. We can extend our work with higher bus system and adding more constraints to our problem. Instead of DC-OPF, ACOPF can be used to solve the power flow problem.

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