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A Mathematical Model for Fluting and Cone Grinding of twist drill

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Abstract: Analytical approach is presented for the inverse problem to manufacture drill flute. Mathematical relation is derived to get different manufacturing parameter for drill flute and cone grinding.

1. Introduction

The profile of the drill flute is more important for the performance of the twist drills it is concern with the cutting angles, chip holding and evacuation, strength and Stability also for dynamic behaviors .Another crucial geometry of the twist drill is “Drill point” which is generated using grinding process. As the geometry of the twist drill is complex, it is very important to understand the modeling of both drill flute and drill point. This paper contains geometric and mathematical modeling of the twist drill.

2. Modeling of drill flute

Accurate geometric model is required to determine grinding wheel profile to generate flute. Basically there are two problems.

a) Determination of flute profile for the given wheel profile.

b) Selection of grinding wheel for the particular drill profile. These are called direct and indirect problem respectively.

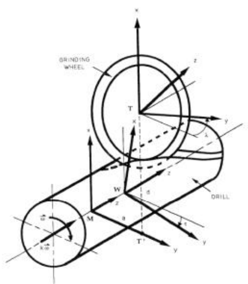


Fig.1.kinematics of drill flute grinding process [1]

2.1 Kinematics of drill flute grinding operation.

Fig 1 indicates the space relationship and relative motion between drill grinding wheels. Let the T -tool frame used to define geometry of grinding wheel.

W -work frame

M-stationary machine frame

Fig 1 shows that origin of the tool frame offset by a and axis distance by d, z axis is attached to the axis of grinding wheel.

2.2 Grinding wheel

The tools which are used to generate the flute of drill those are nothing but a surface of revolution. That is represented by parametric equation. Parametric equation can be generated by revolution of curve which represents the cross section of wheel profile.

If cross-section given by vector

$$T_{p(u)} = [p_1(u), 0, p_2(u)]^T$$

Then the surface is given by

$$T_{p(u,v)} = [p_1(u)cosv, p_2(u)cosv, p_2(u)]^T$$

2.3 Flute Surface definition

An expression for the flute can be defined in the frame w by defining the cross section of drill in xy plane.

$$w_{p(u)} = [f_1(u), f_2(u), 0]^T$$

By rotating and translating in the z axis we can get surface of drill.

$$w_{p(u)} = \begin{bmatrix} \cos v & -\sin v & 0 & 0 \\ \sin v & \cos v & 0 & 0 \\ 0 & 0 & 1 & kv \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} f_1(u) \\ f_2(u) \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} f_1(u)\cos v - \sin v f_2(u) \\ f_1(u)\sin v - \cos v f_2(u) \\ kv \\ 1 \end{bmatrix} \text{..eq. no 1}$$

Fundamental relation between wheel and flute

It says that common normal at the point of contact must intersect the tool axis. Wheel have to move downward, so dot product of the vector that represent the distance and tool axis vector should be zero. Distance move by the wheel for the contact in vector form is:

Axis of tool is:

$$M_{p_n} - M_{p_a}$$

$$M_f \; x \; M_n$$

$M_{p_n} = [M_X, M_Y, M_Z]^T$, position vector on common normal.

$M_{p_a} = [d, a, 0]^T$, position vector on grinding wheel.

$M_f = [0, sin\lambda, cos\lambda]^T$, vector defining grinding wheel axis

$M_n = [M_{n_X}, M_{n_y}, M_{n_z}]^T$, common normal vector

So fundamental relationship should be,

$(M_{p_n} - M_{p_a}) \cdot (M_f \; x \; M_n) = 0$…..eq. no 2

For the simplicity, If the machine frame and work frame are coincide than eq. no 1 is same for the both frame. So common normal to the flute surface

$$M_n = \frac{\partial}{\partial u}(w_{p(u,v)}) \times \frac{\partial}{\partial v}(w_{p(u,v)})$$

$$= \begin{bmatrix} k\dot{f}_1(u)\sin v + k\dot{f}_2(u)\cos v \\ -k\dot{f}_1(u)\cos v + k\dot{f}_2(u)\sin v \\ \dot{f}_1(u)f_1(u) + \dot{f}_2(u)f_2(u) \end{bmatrix}$$

Now after evaluating eq. no. 2 we can get the nonlinear equation in form:

$$(A\sin v + B\cos v + C) + (E\sin v + F\cos v)v = 0$$

…..eq. no 3

Where,

$A = \sin\lambda\, f_2(u)G - k\cos\lambda\,(a\dot{f}_1(u) - d\dot{f}_2(u))$

$B = -\sin\lambda\, f_1(u)G - k\cos\lambda\,(d\dot{f}_1(u) - a\dot{f}_2(u))$

$C = G(d\sin\lambda + k\cos\lambda)$

$E = k^2\sin\lambda\,\dot{f}_1(u)$

$F = k^2\sin\lambda\,\dot{f}_2(u)$

$G = \dot{f}_1(u)f_1(u) + \dot{f}_2(u)f_2(u)$

3. Tool Profile

By solving the eq. no 3 for the particular value of v. We can get the vector in the eq. no. 1then Translate the vector By translate in x and y direction and then rotate by wheel angle with respect to x axis. So we get,

$$T_{p(u,v)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\lambda & \sin\lambda \\ 0 & -\sin\lambda & \cos\lambda \end{bmatrix} \times \begin{bmatrix} M_x - d \\ M_y - a \\ M_z \end{bmatrix}$$

$$= \begin{bmatrix} M_x - d \\ \cos\lambda(M_y - a) + M_z\sin\lambda \\ -\cos\lambda(M_y - a) + M_z\cos\lambda \end{bmatrix}$$

Evaluation of a wheel profile for drill having straight cutting edge

Galloway [2] was the first who derive the definition for surface which must be satisfied for the generation of the straight cutting edge. Then the equations are reproducing by K.F.Ahmaan the equations are:

$$X = \frac{W\cos v}{2\sin\emptyset}$$

$$Y = \frac{-W\sin v}{2\sin\emptyset}$$

Where,

$$v = \emptyset + \frac{W tan \propto \cot\emptyset\cot p}{d}$$

Where is

$$\sin^{-1}\frac{2W}{d} \leq \emptyset \leq \frac{\pi}{2}$$

By solving we can get points and profile of flute, as shown in fig 2a and 2b.

ɸ	v	X	Y
0.411517	0.386711	4.630772	5.285354
0.498737	0.47886	3.711017	2.340213
0.585957	0.569646	3.045559	-0.35557
0.673177	0.659601	2.534952	-1.86067
0.760397	0.749015	2.125218	-2.49723
0.847617	0.838061	1.784449	-2.62936
0.934837	0.926844	1.492503	-2.49319
1.022057	1.015438	1.235955	-2.22302
1.109277	1.103893	1.005439	-1.89122
1.196497	1.192244	0.794134	-1.5352
1.283717	1.280521	0.596859	-1.17317
1.370937	1.368744	0.409512	-0.81286
1.458157	1.456932	0.228686	-0.45635
1.545377	1.545102	0.0514	-0.10279
1.632597	1.633267	-0.1251	0.250033

Fig.2 (a) coordinates for flute profile

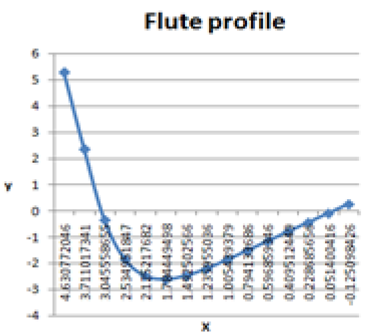


Fig.2 (b) Flute profile

3.1 Model for drill point

It is well known that life of drill affected by the drill point design, by some experiments it is possible to optimize the drill point geometry and life has been found to be increased. Drill point is concern with the chisel edge angle, point angle etc. So it is very important to understand the model of drill point. Basic process of cone cutting contain grinding wheel located at particular angle with the some distance and offset from the drill axis and drill point.

3.2 Mathematical model

It is based on the surface representation of drill flank configuration. It also contains coordinate transformation so that the desired position and direction of the drill or Grinding wheel achieved.

Quadratic equation for the cone surface given by

$$\frac{X'^2}{a^2} + \frac{Y'^2}{b^2} - \frac{Z'^2}{c^2}$$

When the drill point is grinding the drill center moves in the X'-Y' plane hence we can set a =b.

$$\frac{X'^2}{a^2} + \frac{Y'^2}{a^2} = \frac{Z'^2}{c^2} \dots \dots \text{eq. no } 4$$

When a and c approach to zero it becomes

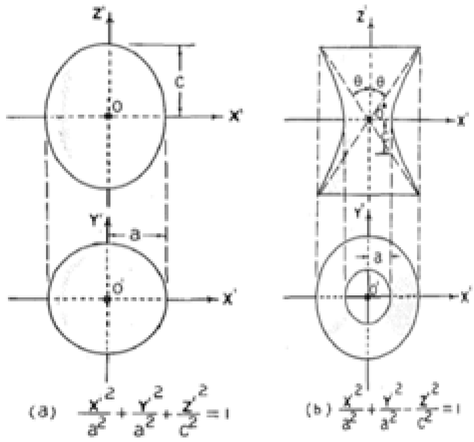
$$\frac{X'^2}{a^2} = (Z' \tan\theta)^2 - Y'^2 \quad \dots \text{eq. no } 5$$

Eq. no. 4 represent the two cone connected at point but we will consider the below cone for the study of drill point. The cone angle is 2θ.In addition to parameter a and c drill flank also depend on S,D,φ which determine the location of drill point and direction of drill w.r.t quadratic grinding surface.

3.3 Coordinate transformation

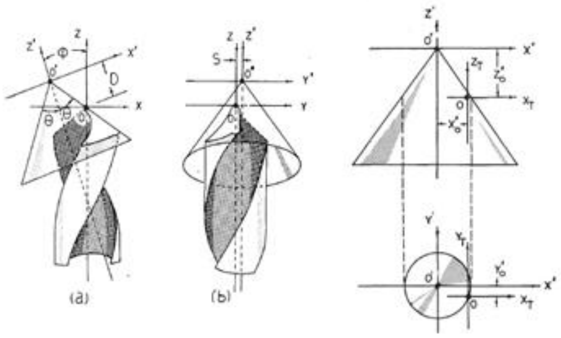
Fig 4 shows the coordinate define for the quadratic surface and drill.

(X, Y, Z) is for drill and (X', Y', Z') is for surface. Coordinate system (X, Y, Z) is selected such that the Z axis is coincide with the drill axis and X axis is parallel to the projection of the straight cutting edge on the plane perpendicular to the Z axis. Eq. no. 4



Eq. no. 4 is define for the coordinate system (X',Y',Z') so that have to transfer to the drill coordinate by translation and rotation of the origin and axis respectively.

Translation of the origin



Assume in the (x',y',z') coordinate system ,the drill center point O is located at the X'_0, Y'_0, Z'_0

Let,

X'_0= -S

Y'_0= D

From eq. no 5

$$X'_{0}=\sqrt{(Z'_{0}\tan\theta)^{2}-Y'_{0}{}^{2}}$$

by translate the(X',Y',Z') using above coordinates we get,

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} X_T \\ Y_T \\ Z_T \end{bmatrix} + \begin{bmatrix} \sqrt{(S\tan\theta)^2 - D^2} \\ -S \\ D \end{bmatrix} \text{.....eq. no 6}$$

Where X^T, Y^T, Z^T denote new translated coordinate system.

4. Rotation of coordinate

These can be done by rotating the coordinate in X-Z plane by angle ϕ. So rotation
Matrix is given by:

$$\begin{bmatrix} X_T \\ Y_T \\ Z_T \end{bmatrix} = \begin{bmatrix} \cos\emptyset & 0 & \sin\emptyset \\ 0 & 1 & 0 \\ -\sin\emptyset & 0 & \cos\emptyset \end{bmatrix} + \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \text{.....eq. no 7}$$

by combining eq. no 6 and 7

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} X\ \cos\emptyset + Z\sin\emptyset + \sqrt{(S\tan\theta)^2 - D^2} \\ Y - S \\ -X\sin\emptyset + Z\cos\emptyset + D \end{bmatrix}$$
...eq. no 8

Evaluating eq. no. 5 using eq. no 7 we get model for the conical drill.

$$(X\ \cos\emptyset + Z\sin\emptyset + \sqrt{(S\tan\theta)^2 - D^2})^2 + (Y-S)^2 + (-X\sin\emptyset + Z\cos\emptyset + D)^2 tan^2\theta = 0$$

Solving the above nonlinear equation one can find value for S and D. also from the fig no.3

$$\emptyset + \theta = p$$

Where p is half point angle.

Conclusion

Using these Mathematical equations one can create application program so that modeling of drill bit become easy and fast in CAD software and this gives an advantage of saving time as well provide the result of actual model analysis before the production of drill.

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